

Geometry of Information Propagation in Massively Dense Ad hoc Networks

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ABSTRACT

Using the fact that effective wireless range decreases in inverse function of local traffic density, we show that a variable traffic density impacts the curvature of paths in a dense wireless ad hoc network the same way a variable optical density bends light paths. We set up the general laws that paths must satisfy in presence of traffic flow density. We give some example of tractable network traffic topologies.

Categories and Subject Descriptors

C.2.2 [Network Protocols]: Routing protocols
; H.1.1 [Systems and Information Theory]: information theory

General Terms

Performance

1. INTRODUCTION

Mobile ad hoc networks involve nodes that are moving on a network domain and communicate via radio means. The domain of a network can be indifferently a battlefield, a urban quarter, a building floor, etc. Most papers in the litterature take as an assumption models where the distribution of traffic and nodes are uniform over their network domain. This basic model leads to fundamental results, as the illuminating result of Gupta and Kumar [2] which states that the maximum capacity per node in a flat domain is in $O(\sqrt{\frac{1}{N \log N}})$. In this paper we will depart from the uniform model and assume that the traffic density varies with node location. We will provide results on how shortest path are affected by traffic density gradients. In particular we will show that in asymptotic conditions the routing paths obey to similar laws as in non linear optic.

In their reference paper on the capacity of mobile ad hoc

networks, Gupta and Kumar [2] showed that in presence of traffic density of λ bit per time unit per square unit area, the typical radius of correct reception decays in $O(\frac{1}{\sqrt{\lambda}})$. This result assumes an uniform density model and λ is supposed to be the density of traffic including the load generated by packet that are retransmitted on their way to their destination on multihop paths. To our view, this estimate is the most fundamental result. As a direct consequence the average number of hops needed to connect two arbitrary points in a bounded domain is therefore $O(\sqrt{\lambda})$ since the distance must be divided by the radio ranges. As pointed out by Gupta and Kumar, this property has a strong implication in the evaluation of the maximum capacity attainable by a random node when the node density increases. If C is the capacity generated by each node and N is the number of nodes in the network, Gupta and Kumar found that the maximum bandwidth attainable is $C_N = \frac{\alpha}{\sqrt{N \log N}}$, quantity α depending on propagation models. However the order of magnitude is easy to get: the density of traffic generated per unit square are is $O(CN)$. Let $r(C, N)$ be the typical radio range, thus the number of retransmissions needed to route a packet from its source to its destination is $O(\frac{1}{r(C, N)})$. The latter estimate, in turn, yields a traffic density (including retransmissions) of $O(\frac{CN}{r(C, N)})$. Therefore $r(C, N) = O(\sqrt{\frac{r(C, N)}{CN}})$, namely $r(C, N) = O((CN)^{-1})$. The average number of neighbor per node is $O(\pi r(C, N)^2 N)$; it should be larger than $\log N$ in order to guarantee connectivity, which leads to the estimate $C = O((N \log N)^{-1/2})$.

If instead we consider that the network spans on a domain in dimension D , then the radius of correct reception decays in the inverse of the D th root of emitter density, which impacts the maximum capacity (replacing the exponent $-1/2$ by $-1/D$).

This paper addresses the case where the traffic pattern is not uniform but varies as a continuous function of node location. We investigate the case where the traffic and node densities are large enough to make the efficient radio ranges infinitely small (compared to traffic density gradient and domain size). In this perspective, shortest paths (in number of hops) from sources to destinations look like continuous lines. In the sequel we call lines generated by shortest path routes, *propagation lines*. In the uniform model propagation lines are expected to be straight lines. In this paper we limit our investigation to the propagation lines and we analyze how the latter are affected by the variation of traffic density. We

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summarize our findings into macroscopic differential equations involving propagation lines curvatures.

The paper is organized as follows. The second section investigates more thoroughly a model of mobile ad hoc network in order to provide more accurate estimates about Gupta and Kumar results. The model assumes a 2D domain under slotted time with a density of λ emitters per slot and square area unit. Under this simple model we will provide quantitative confirmation of Gupta and Kumar results. The third section introduces the concept of massively dense networks and how propagation lines are affected by variable traffic density. At a macroscopic level the variable traffic density acts like a variable optical index and curves the propagation lines from sources to destinations. The fourth section describes some elementary case where the distribution of shortest path can be exactly determined. In general the problem is very difficult since the concentration of route affect traffic density and in return affect the route path and this is expressed via non trivial differential equations.

2. QUANTITATIVE RESULTS ON TIME SLOTTED NETWORKS

2.1 Quantification of the problem

Kumar and Gupta estimates were originally derived from information theory considerations and are not related to any particular network implementation. If we assume a specific implementation, then there will be a quantity β such that the typical radius of correct reception of a packet is equal to $\frac{\beta}{\sqrt{\lambda}}$. By typical radius we mean the radius below which probability of correct reception of a packet is above a given threshold. The quantity β will depend on many parameters such as the probability threshold, the attenuation coefficient of wave propagation and the minimum signal-over-noise ratio required for correct reception. Notice that the typical disk of correct reception contains in average a finite number of transmitters per slot, since the area is proportional to $\frac{1}{\lambda}$.

If we consider a network dispatched in a domain of dimension D then the estimate of the radius will be $\frac{\beta}{\lambda^{1/D}}$. In the sequel we will look at 2D domains generalizing occasionally the results on other dimensions.

When the density λ increases in a fixed domain, then the minimum number of hops connecting two points A, B tends to be equivalent to $d(A, B) \frac{\sqrt{\lambda}}{\beta}$ where $d(A, B)$ denotes the euclidian distance between mobile node A and mobile node B . Meanwhile, the increase of the number of relays naturally increases the traffic density. If ν is the actual traffic density generation per unit area, *i.e.* the traffic locally generated on mobile nodes, not the traffic relayed by the mobile nodes, then the average density traffic will satisfy the identity: $\lambda = \nu \bar{d} \frac{\sqrt{\lambda}}{\beta}$ where \bar{d} is the average euclidian distance between two end points in a connection.

This previous identity assumes that the pattern of path between points covers the domain in an uniform manner so that the traffic density, generated and relayed, is constant on the whole domain. In this case the path that connect two points with the minimum number of hops is very close to the straight line. But the question arises about the shape of the shortest path when the traffic density is not uniform. We will show that when the density increases while keeping proportional to a given continuous function, then the prop-

agation paths tend to conform to continuous line, that we call propagation lines. Under these assumptions we will provide the general equations that the propagation lines must satisfy. We will show that variable traffic densities affect shortest path the same way as variable optical indices affect light path in a physical medium.

2.2 Propagation model

We consider the following model: time is slotted, all mobile nodes are synchronized, transmissions on beginning of slots. We consider an area of arbitrary size \mathcal{A} (we will ignore border effect). N transmitters are distributed according to a Poisson process. We call λ the density of transmitter per slot and per square area unit. We have $\lambda = fN/\mathcal{A}$ where f is the rate of packets transmission per slot and per node.

Let a node X at a random position (we ignore border effects). We assume that all nodes transmit at the same nominal power. The reception signal at distance r is $P(r) = r^{-\alpha}$ with $\alpha > 2$. Typically $\alpha = 2.5$. Let W the signal intensity received by node X at a random slot. Quantity W is a random variable: let $w(x)$ its density function. In [1] it is shown that the Laplace transform of $w(x)$, $\tilde{w}(\theta) = \int w(x)e^{-x\theta} dx$ satisfies the identity:

$$\tilde{w}(\theta) = \exp(2\pi\lambda \int_0^\infty (e^{-\theta r^{-\alpha}} - 1) r dr). \quad (1)$$

Using standard algebra we get

$$\tilde{w}(\theta) = \exp(-\lambda\pi\Gamma(1 - \frac{2}{\alpha})\theta^{2/\alpha}) \quad (2)$$

If the node location domain was a line instead of an area (consider a sequence of mobiles nodes on a road) then we would have

$$\tilde{w}(\theta) = \exp(-\lambda\Gamma(1 - \frac{1}{\alpha})\theta^{1/\alpha}) \quad (3)$$

If, instead the location domain was a volume (consider aircrafts network), then

$$\tilde{w}(\theta) = \exp(-\frac{4}{3}\lambda\pi\Gamma(1 - \frac{3}{\alpha})\theta^{3/\alpha}) \quad (4)$$

In the following we restrict ourselves on a 2D domain.

2.3 Neighbor model

A node is considered neighbor of another node if the probability of receiving packets from each other is greater than a certain threshold p_0 . For example $p_0 = 1/3$. Under this model, we can affect to p_0 the value which optimizes the distance traveled by a packet per transmission as in [3]. We assume that the slotted system contains an acknowledgment mechanism so that each succesful transmission does not trigger any new retransmission for the same hop. In this case the distance travelled by the packet is equal to the distance from the transmitter to the receiver. When the transmission fails then the distance is zero and the node reschedule a new retransmission at a random time (we assume that λ involve the load due to retransmissions).

We assume that a packet can be decoded if its signal-over-noise is greater than a given threshold K . Typically $K = 10$. Therefore another node is neighbor if its distance r is such that $P(W < r^{-\alpha}/K) > p_0$, *i.e.* when $r < r(\lambda)$ where $r(\lambda)$ is the critical radius such that $\int_0^{r(\lambda)} w(x)dx = p_0$. By simple algebra it comes that $r(\lambda) = \lambda^{-1/2}r(1)$. This result confirms the result of Gupta and Kumar in this very specific

network model. We find $\beta = r(1)$. The surface covered by the radius $r(\lambda)$ is the neighborhood area $\sigma(\lambda) = \frac{\sigma(1)}{\lambda}$ and $\sigma(1) = \pi r(1)^2$.

2.4 Computation of β , $r(1)$ and $\sigma(1)$

In order to simplify the derivations we set $C = \pi\Gamma(1 - \frac{2}{\alpha})$ and $\gamma = \frac{2}{\alpha}$. By application of reverse Laplace we have:

$$P(W < x) = \frac{1}{2i\pi} \int_{-i\infty}^{+i\infty} \frac{\tilde{w}(\theta)}{\theta} e^{\theta x} d\theta \quad (5)$$

Expanding $\tilde{w}(\theta) = \sum_n \frac{(-C)^n}{n!} \theta^{n\gamma}$, we get

$$P(W < x) = \frac{1}{2i\pi} \sum_n \frac{(-C)^n}{n!} \int_{-i\infty}^{+i\infty} \theta^{n\gamma-1} e^{\theta x} d\theta \quad (6)$$

By bending the integration path toward the negative axis we get

$$\begin{aligned} \frac{1}{2i\pi} \int_{-i\infty}^{+i\infty} \theta^{n\gamma-1} e^{\theta x} d\theta &= \frac{\sin(\pi n\gamma)}{\pi} \int_0^{\infty} \theta^{n\gamma-1} e^{-\theta x} d\theta \\ &= \frac{\sin(\pi n\gamma)}{\pi} \Gamma(n\gamma) x^{-n\gamma} \end{aligned}$$

Figure 1 shows the plot of $P(W < x)$ versus x for $\alpha = 2.5$ and $\lambda = 1$. Notice that $P(W < x)$ reaches $p_0 = 1/3$ close to $x = x_0 = 20$. Therefore $\beta = r(1) = (x_0 K)^{-1/\alpha} \approx 0.12$. Therefore $\sigma(1) \approx 0.045$.

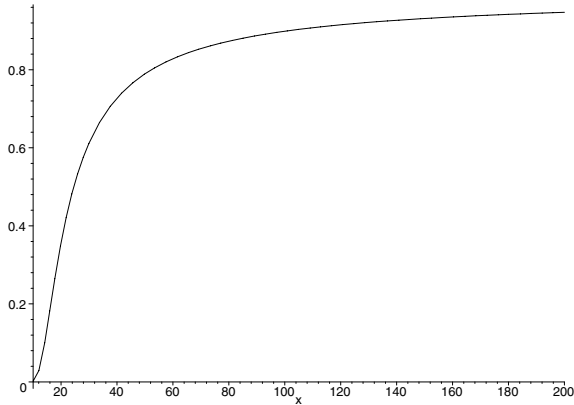


Figure 1: Quantity $P(W < x)$ versus x for $\alpha = 2.5$, no fading.

2.5 Modelization of fading

Signals propagating through random obstacles experience random fadings. An usual modelization of fading consists into introducing a random factor F to signal attenuation at distance r : $r^{-\alpha}$. For example $\log F$ is uniform on $[-v, v]$. In this case we have a new expression of $\tilde{w}(\theta)$:

$$\tilde{w}(\theta) = \exp(-\pi\lambda\Gamma(1 - \frac{2}{\alpha})\phi(-\frac{2}{\alpha})\theta^{2/\alpha}) \quad (7)$$

with $\phi(s) = E(F^{-s})$, the Dirichlet transform of the fading. When fading is uniform on $[-v, v]$ we have $\phi(s) = \frac{\sinh(sv)}{sv}$.

For any given real number x we also have $P(W < xF)$ equaling

$$\sum_n \frac{(-CF(-\gamma))^n \sin(\pi n\gamma)}{\pi n!} \Gamma(n\gamma) \phi(n\gamma) x^{-n\gamma} \quad (8)$$

which helps the computation of $\sigma(1)$ with fading.

3. MASSIVELY DENSE NETWORKS

We now consider massively dense networks on a 2D domain. We denote by $\lambda(x, y)$ the traffic density at the point of coordinate (x, y) on the domain. We suppose that function $\lambda(x, y)$ is continuous in (x, y) , or at least Lebesgue integrable. When $\lambda(x, y)$ are uniformly large, the results of Gupta Kumar together with the result of the previous section state that the radio ranges tend to be “microscopic” and routes can be considered as continuous lines between nodes. Packets travelling on a route \mathcal{C} passing on the point of coordinate (x, y) will experience hops of length $\frac{\beta}{\sqrt{\lambda(x, y)}}$ passing in the vicinity of point (x, y) . Let $n(x, y) = \frac{\sqrt{\lambda(x, y)}}{\beta}$. The number of hops that a packet will experience on route \mathcal{C} is something close to $\int_{\mathcal{C}} n(x(s), y(s)) ds$ where s is a curvilinear abscissa on route \mathcal{C} .

In the sequel we are looking for route with the shortest hop number. Searching the path that minimize the hop number between two points A and B is therefore equivalent for looking for the path light between A and B in a medium with non-uniform optical index $\lambda(x, y)$. It is known that the optimal path satisfies on each of its point $\mathbf{z}(s) = (x(s), y(s))$:

$$\frac{d}{ds}(n(\mathbf{z}(s)) \frac{d\mathbf{z}(s)}{ds}) = \nabla n(\mathbf{z}(s)) \quad (9)$$

where ∇ is symbol of gradient vector. Therefore finding the optimal path is just an application of geometric optics. Notice that when $\nabla n = 0$ (uniform traffic density) propagation lines are straight lines (no curvature).

However we face a major problem in the fact that the distribution of path is actually impacting traffic density. This lead to an egg-and-chicken problem which may not that easy to solve. We call $\Phi(x, y)$ the flow density of information transiting in the vicinity of point (x, y) . Quantity $\Phi(x, y)$ is expressed in bit per meter, since it express the flow of packet crossing a virtual unit of segment of length of 1 meter centered on point (x, y) . This flow impact the traffic density by the fact that each packet must be relayed every $\beta/\sqrt{\lambda(x, y)}$ meter in the vicinity of point (x, y) . Therefore locally:

$$\lambda(x, y) = \Phi(x, y) \frac{\sqrt{\lambda(x, y)}}{\beta} \quad (10)$$

In other words $\lambda(x, y) = (\frac{\Phi(x, y)}{\beta})^2$ and

$$n(x, y) = \frac{\Phi(x, y)}{\beta^2} \quad (11)$$

When considering domain of dimension D we have $\lambda = \Phi \frac{\lambda^{1/D}}{\beta}$ and $n = (\frac{\Phi}{\beta^D})^{\frac{1}{D-1}}$. Notice that the equations are singular when $D = 1$.

As an example we can assume a planar domain massively and uniformly filled with mobile nodes and gateway nodes. We denote by μ_G the spatial density of gateways. We assume that the mobile nodes are much more dense than

the gateways. We denote by ν the traffic density generated in any point. ν is expressed in bits per square meters per slot. The flow density Φ is constant in the domain and is equal to $\nu\bar{d}$. We suppose that mobile nodes send and receive flows from their closest gateway. Therefore $\bar{d} = \int_0^\infty \exp(-\pi\mu_G r^2) dr = \frac{1}{2\sqrt{\mu_G}}$:

$$\Phi = \frac{\nu}{2\sqrt{\mu_G}} \quad (12)$$

But in this case $\nabla n = 0$ and propagation lines are straight lines.

4. EXAMPLES OF TRACTABLE TRAFFIC TOPOLOGIES WITH CURVED PROPAGATION PATHS

As a general framework we rewrite equation (9) in

$$n \frac{d}{ds} \left(n \frac{d\mathbf{z}}{ds} \right) = \frac{\nabla n^2}{2} \quad (13)$$

This equation has a straightforward analogy with dynamics. It turns out that if a particle were moving at speed $n(\mathbf{z})$ on every point (\mathbf{z}) , then traveling on an optimal path would imply that the particle would be submitted to an acceleration $\frac{\nabla n^2(\mathbf{z})}{2}$, or, in other words, would be submitted to a potential $\frac{n^2(\mathbf{z})}{2}$.

4.1 The central gateway model

We suppose that the domain is a disk of radius R where nodes on the border maintain permanent and identical flows to a gateway located in the center of the disk. Our aim is to describe the shortest path between two points A and B which are inside the disk. We do suppose that the flow between nodes A and B is small enough that it doesn't significantly affect the global traffic density. Without this assumption the problem will be very difficult to handle.

Nodes on the outer circle transmit data toward the central gateway (or equivalently receive data from the central gateway). By symmetry considerations, the shortest path between the gateway and any node on the circle just follows the radius drawn from the disk center to this node. Thus we have the identity:

$$\Phi(\mathbf{z}) = \ell \frac{R}{|\mathbf{z}|} \quad (14)$$

where ℓ is the linear density of traffic generated on the circle, expressed in bit per meter per slot. We notice that there is a singularity when $\mathbf{z} = 0$. We can cope with this singularity by removing from the domain a disk of smaller radius $r < R$, and by imagining that the inner circle contains gateways uniformly distributed. Nodes would access their closest gateway and their path would still head to the center of the disk.

If we take the analogy with dynamics, the optimal path between two points is the trajectory of a moving body at speed $|\dot{\mathbf{z}}| = n(\mathbf{z}) = \frac{\ell R}{\beta^2 |\mathbf{z}|}$ and acceleration $\ddot{\mathbf{z}} = \frac{1}{2} \nabla n^2(\mathbf{z}) = -\left(\frac{\ell}{\beta^2 |\mathbf{z}|^2}\right)^2 \mathbf{z}$. Since the acceleration is central, *i.e.* is colinear with the vector \mathbf{z} , Kepler's law of area conservation applies: The area of the triangle made by vector \mathbf{z} and vector $\dot{\mathbf{z}}$ is constant on the body trajectory. Since this area is equal to $\frac{\ell \sin \theta}{\beta^2}$ where θ is the angle between radius vector and speed

vector. It turns out from Kepler's law that θ is constant and we can state the theorem:

THEOREM 1. *The optimal path between two points A and B in the central gateway configuration is the portion of logarithmic spiral centered on the gateway that connects the two points.*

COROLLARY 1. *When the two points are at same distance to the gateway, then the optimal path is the portion of circle centered on the gateway connecting the two points.*

The consequence of this is that the optimal path is never the straight line and when the points are opposite with respect to the central gateway, then the optimal path starts orthogonally to the linear segment connecting the points.

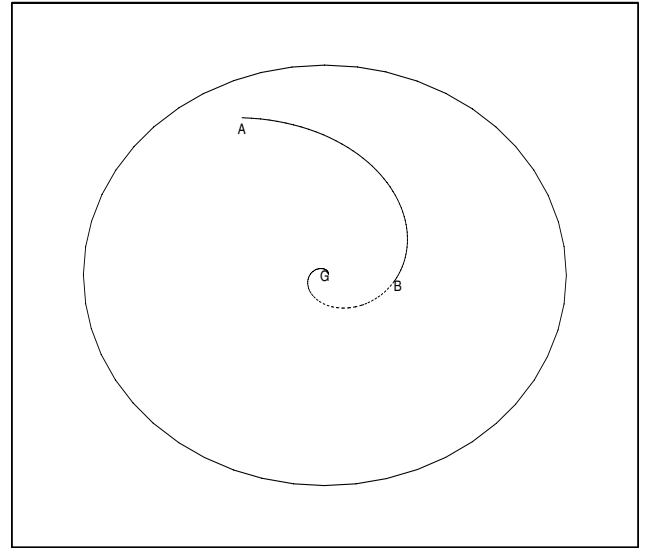


Figure 2: Optimal path between points A and B in the central gateway configuration.

4.2 The border gateways model

We take the same disk domain as in the previous example. Now we assume that the gateways are densely and uniformly distributed on the border of the outer circle and all nodes generate uniform traffic toward their closest gateway, *i.e.* the gateway which is on the radius of the circle that contains the node. Due to symmetry consideration, the shortest path to the gateway is the radius segment. Thus easy algebra gives

$$\Phi(\mathbf{z}) = \ell \frac{|\mathbf{z}|}{R} \quad (15)$$

where ℓ is the linear density of information arriving on the outer circle. Taking the analogy with the body in motion, an optimal path is the trajectory of the body in motion such that $|\dot{\mathbf{z}}| = \gamma |\mathbf{z}|$ and $\ddot{\mathbf{z}} = \gamma^2 \mathbf{z}$ with $\gamma = \frac{\ell}{R\beta^2}$.

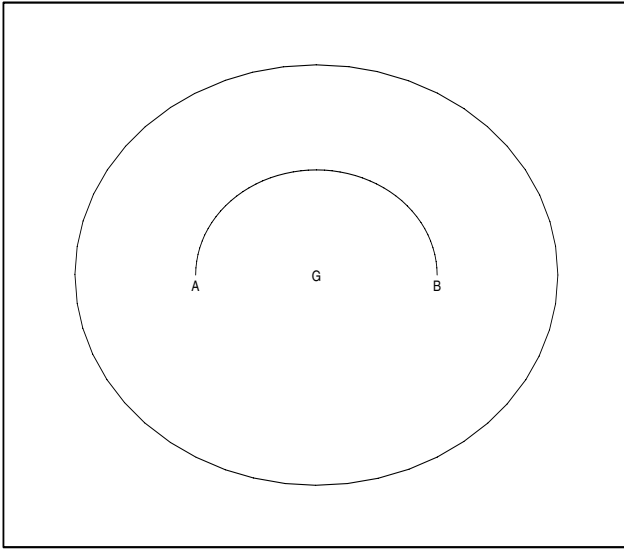


Figure 3: Optimal path between two opposite points A and B in the central gateway configuration.

The solution of the differential equation on time variable t is of the form $\mathbf{z} = e^{\gamma t} \mathbf{u} + e^{-\gamma t} \mathbf{v}$ where \mathbf{u} and \mathbf{v} are arbitrary vectors. The identity $|\dot{\mathbf{z}}| = \gamma |\mathbf{z}|$ implies that the vectors \mathbf{u} and \mathbf{v} must be orthogonal. Consequently an optimal path must be a branch of hyperboloid. However a branch of hiperbol can only connect nodes A and B that form an angle smaller than $\frac{\pi}{2}$ with the disk center. When the angle is larger then the shortest path is simply made of the union of the two segments connecting the center with the nodes A and B .

THEOREM 2. *In the border gateway configuration on a disk of center O , the optimal path between two nodes A and B is the following:*

- (i) *When the angle in O of the triangle (A, B, O) is less than $\frac{\pi}{2}$, then the optimal path is the branch of hiperbol that connect the two points A and B ;*
- (ii) *When the angle in O of the triangle (A, B, O) is larger than or equal to $\frac{\pi}{2}$, then the optimal path is the union of the segments (A, O) and (O, B) .*

5. INFORMATION FLOW TENSOR AND FUTURE WORK

The previous examples are tractable because the path to the gateways were determined *a priori* by symmetry considerations and were not disturbed by the variation of parameter $n(\mathbf{z})$. In fact the problem is that propagation lines and the traffic densities mutually affect each other. We have

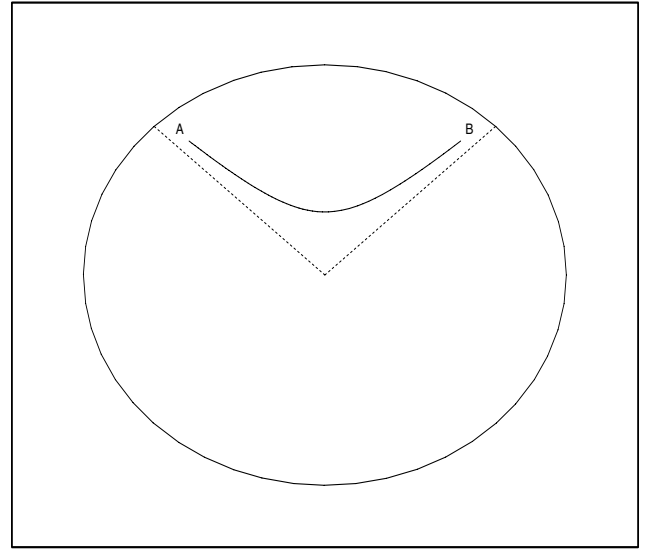


Figure 4: Optimal path between two points A and B with angle with center smaller than $\frac{\pi}{2}$, in the outer gateways configuration.

to describe the information flow in more details. Let consider a point (x, y) , let \mathbf{u}_i be the sequence of information flow vectors passing in the vicinity of this point, each vector is expressed in bit per meter. We define the tensor of information flow vector \mathbf{T} the following:

$$\mathbf{T}(x, y) = \sum_i \frac{1}{|\mathbf{u}_i|} \mathbf{u}_i \otimes \mathbf{u}_i \quad (16)$$

The definition is equivalent to the tensor stress-energy in physics. We have $\Phi = \text{tr}(\mathbf{T}) = \sum_i |\mathbf{u}_i|$. The divergence of \mathbf{T} can be interpreted as local generation of information flow: $\nabla \cdot \mathbf{T} = \mathbf{v}$ where \mathbf{v} is the sum of all information vectors generated in the vivinity of point (x, y) . An information vector \mathbf{v} is when information is generated in volume $|\mathbf{v}|$ on point (x, y) and is transmitted toward a path with initial vector $\frac{\mathbf{v}}{|\mathbf{v}|}$.

It seems also that propagation lines don't change when the route optimization criterium changes. For example if hop number is changed in packet total delay time, the route should basically remains the same. The reason for this conjecture is that the condition of traffic at any given point in the network location is basically the same *modulo* an homothetic factor $\lambda(x, y)$. The only aspect which changes is the distance travelled by the packet per hop, but the delay per hop will be the same in distribution. We have a similar point about bandwidth allocation criterium. The transmission at any point will take the same amount of bandwidth, only the per hop distance travelled by the packet will differ.

Future works should also emphasize the mobility aspect of massively dense networks. Although sensors and captors are

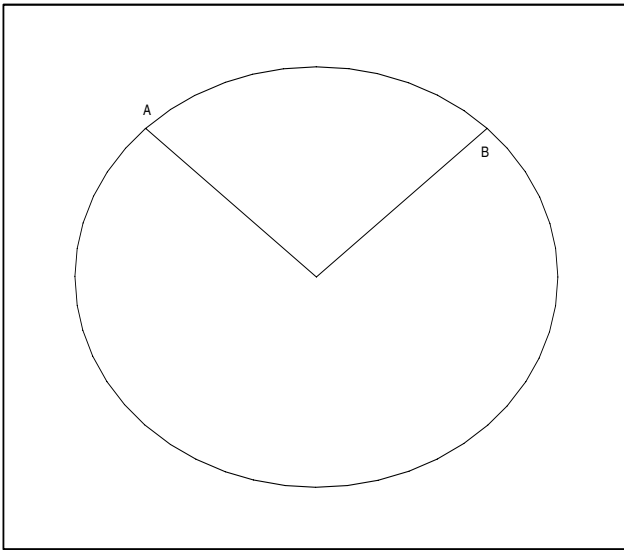


Figure 5: Optimal path between two points A and B with angle with center equal to $\frac{\pi}{2}$, in the outer gateways configuration.

expected to be mobile devices, it will interesting to introduce mobility in the framework. Mobility will affect route path if intermediate nodes are allowed to store packet while moving instead of immediately retransmitting it like an hot potatoe. If one relieves the constraint of delay delivery, then one can considerably increase the capacity per node if the mobility model satisfy some strong properties [4]. It would be interesting to analyse the previous phenomenon in a space-time consideration using the framework presented in this paper.

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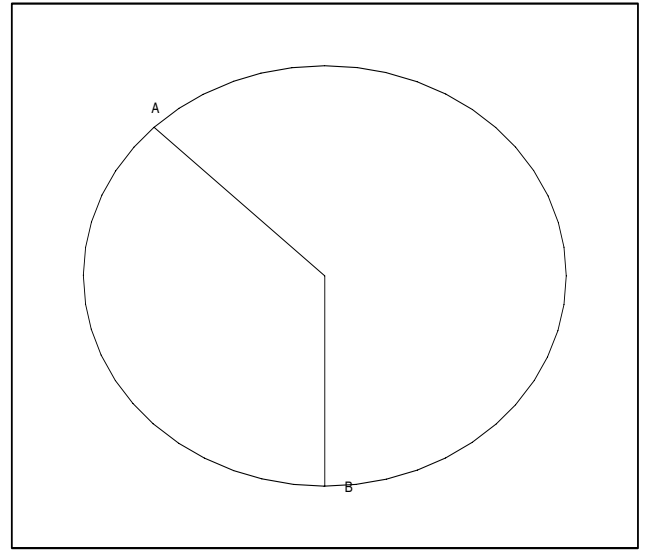


Figure 6: Optimal path between two points A and B with angle with center greater than $\frac{\pi}{2}$, in the outer gateways configuration.