Rate Allocation in Wireless Sensor Networks with Network Lifetime Requirement

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ABSTRACT

An important performance consideration for wireless sensor networks is the amount of information collected by all the nodes in the network over the course of network lifetime. Since the objective of maximizing the sum of rates of all the nodes in the network can lead to a severe bias in rate allocation among the nodes, we advocate the use of *lexicographical max-min* (LMM) rate allocation for the nodes. To calculate the LMM rate allocation vector, we develop a polynomial-time algorithm by exploiting the *parametric analysis* (PA) technique from linear programming (LP), which we call *serial LP with Parametric Analysis* (SLP-PA). We show that the SLP-PA can be also employed to address the so-called LMM node lifetime problem much more efficiently than an existing technique proposed in the literature. More important, we show that there exists an elegant *duality* relationship between the LMM rate allocation problem and the LMM node lifetime problem. Therefore, it is sufficient to solve any one of the two problems and important insights can be obtained by inferring duality results for the other problem.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication

General Terms

Algorithms, Performance, Theory

Keywords

Wireless sensor networks, energy constraint, network capacity, rate allocation, lexicographic max-min, node lifetime, parametric analysis, linear programming, flow routing

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1. INTRODUCTION

Wireless sensor networks consist of battery-powered nodes that are endowed with a multitude of sensing modalities including multimedia (*e.g.,* video, audio) and scalar data (*e.g.,* temperature, pressure, light, magnetometer, infrared). Although there have been significant improvements in processor design and computing, advances in battery technology still lag behind, making energy resource considerations the fundamental challenge in wireless sensor networks. As a consequence, there have been active research efforts on exploring performance limits of wireless sensor networks. These performance limits include, among others, *network capacity* (see e.g., [12]) and *network lifetime* (see e.g., [7, 8]). Network capacity typically refers to the maximum amount of bit volume that can be successfully delivered to the base-station ("sink node") by all the nodes in the network, while network lifetime refers to the maximum time limit that nodes in the network remain alive until one or more nodes drain up their energy.

In this paper, we consider an important overarching problem that encompasses both performance metrics. In particular, we study the network capacity problem under a given network lifetime requirement. Specifically, for a wireless sensor network where each node is provisioned with an initial energy, if all nodes are required to live up to a certain lifetime criterion, what is the maximum amount of bit volume that can be generated by the entire network? At first glance, it appears desirable to maximize the sum of rates from all the nodes in the network, subject to the condition that each node can meet the network lifetime requirement. Mathematically, this problem can be formulated as a linear programming (LP) problem (see Section 2.2) within which the objective function is defined as the sum of rates over all the nodes in the network and the constraints are: (1) flow balance is preserved at each node, and (2) the energy constraint at each node can be met for the given network lifetime requirement. However, the solution to this problem shows (see Section 5) that although the network capacity (i.e., the sum of bit rates over all nodes) is maximized, there exists a severe bias in the rate allocation among the nodes. In particular, those nodes that consume the least amount of power on their data path toward the base-station will be allocated with much more bit rates than other nodes in the network. Consequently, the data collection behavior for the entire network only favors certain nodes that have this property, while other nodes will be unfavorably penalized with much smaller bit rates.

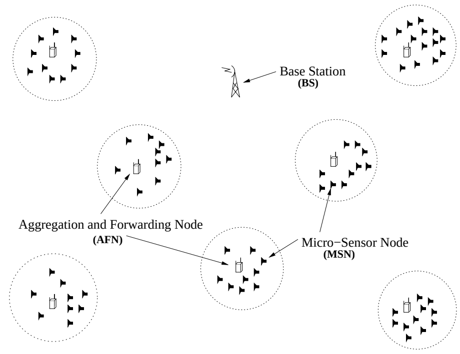
The fairness issue associated with the network capacity maximization objective calls for a careful consideration in the rate allocation among the nodes. In this paper, we investigate the rate allocation problem in an energy-constrained sensor network for a given network lifetime requirement. Our objective is to achieve a certain measure of optimality in the rate allocation that takes into account both fairness and bit rate maximization. We advocate to use of the so-called *Lexicographic Max-Min* (LMM) criterion [14], which maximizes the bit rates for *all* the nodes until one or more nodes reach their energy limit for the given network lifetime requirement. At first level, the smallest rate among all the nodes is maximized. We continue to maximize the second level of smallest rate and so forth. The LMM rate allocation criterion is appealing since it addresses both fairness and efficiency (i.e., bit rate maximization) in an energy-constrained network.

A naive approach to the LMM rate allocation problem would be to apply a max-min-like iterative procedure. Under this naive approach, successive LPs are employed to calculate the maximum rate at each level based on the available energy for the remaining nodes, until all nodes use up their energy. We call this naive approach "serial LP" (SLP). We show that, although SLP appears intuitive, unfortunately it gives an *incorrect* solution. To understand how this could happen, we must understand a fundamental difference between the LMM rate allocation problem described here and the classical max-min rate allocation in [3]. Under the LMM rate allocation problem, the rate allocation problem is implicitly *coupled* with a flow routing problem, while under the classical max-min rate allocation, there is no routing problem involved since the routes for all flows are fixed. As it turns out, for the LMM rate allocation problem, *any iterative rate allocation approach that requires energy reservation at each iteration is incorrect.* This is because, unlike max-min, which addresses only the rate allocation problem with fixed routes and yields a unique solution at each iteration, for the LMM rate allocation problem, starting from the first iteration, there usually exist *non-unique* flow routing solutions corresponding to the same rate allocation at each level. Consequently, each of these flow routing solutions will yield *different* available energy levels on the remaining nodes for future iterations and so forth, leading to a different rate allocation vector, which usually does not coincide with the optimal LMM rate allocation vector.

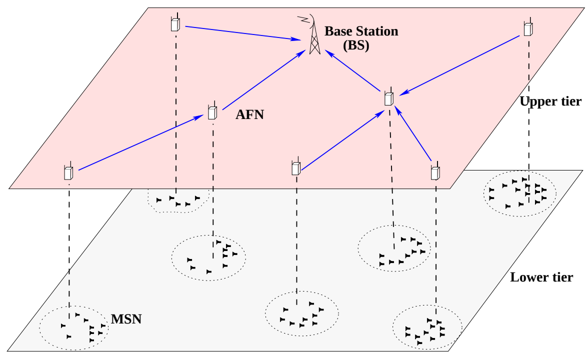
In this paper, we develop an efficient polynomial-time algorithm to solve the LMM rate allocation problem. We exploit the so-called *parametric analysis* (PA) technique [2] at each rate level to determine the minimum set of nodes that must deplete their energy. We call this approach *serial LP with PA* (SLP-PA). In most cases when the problem is non-degenerate, the SLP-PA algorithm is extremely efficient and only requires quadratic time complexity in determining the minimum node set for each rate level. Even for the rare case when the problem is degenerate, the SLP-PA algorithm is still much more efficient than the slack variable (SV)-based approach proposed in [6], due to fewer number of LPs involved at each rate level.

We also extend the PA technique for the LMM rate allocation problem to address the so-called maximum node lifetime curve problem in [6], which we call LMM node lifetime problem. We show that the SLP-PA approach is much more efficient than the slack variable (SV)-based approach described in [6]. More importantly, we show that there exists a simple and elegant *duality* relationship between the LMM rate allocation problem and the LMM node lifetime problem. As a result, it is sufficient to solve only one of these two problems and important insights can be obtained by inferring duality results for the other problem.

The remainder of this paper is organized as follows. In Section 2, we describe the network and energy model, and formulate the LMM rate allocation problem. Section 3 presents our SLP-PA algorithm to the LMM rate allocation problem. In Section 4, we introduce the LMM node lifetime problem and apply the SLP-PA algorithm to solve it. We also show how the LMM rate allocation problem and the LMM node lifetime problem are linked by a duality relationship. Numerical results are presented in Section 5. Section 6 reviews related work and Section 7 concludes this paper.



(a) Physical topology.



(b) A hierarchical view.

Figure 1: Reference architecture for two-tiered wireless sensor networks.

2. SYSTEM MODELING AND PROBLEM FORMULATION

We consider a two-tiered architecture for wireless sensor networks. The two-tiered network architecture is motivated by recent advances in *distributed source coding* (DSC) [9, 15, 17], which is capable of removing redundancy in information collected among neighboring sensors without inter-sensor communications. Figures 1(a) and (b) show the *physical* and *hierarchical* network topology for such a network, respectively. There are three types of nodes in the network, namely, *micro-sensor nodes* (MSNs), *aggregation and forwarding nodes* (AFNs), and a *base-station* (BS). The MSNs can be application-specific sensor nodes (*e.g.*, temperature sensor nodes (TSNs), pressure sensor nodes (PSNs), and video sensor nodes (VSNs)) and they constitute the lower tier of the network. They are deployed in groups (or clusters) at strategic locations for surveillance or monitoring applications. The MSNs are small and low-cost. The objective of an MSN is very simple: Once triggered by an event (*e.g.,* detection of motion or biological/chemical agents), it starts to capture live information (*e.g.,* video), which it sends directly to the local AFN.[1]

For each cluster of MSNs, there is one AFN, which is different from an MSN in terms of physical properties and functions. The primary functions of an AFN are: (1) *data aggregation* (or "fusion") for data flows from the local cluster of MSNs, and (2) *forwarding* (or relaying) the aggregated information to the next hop AFN (toward the base-station). For data fusion, an AFN analyzes the content of each data stream (*e.g.,* video) it receives, from which it composes a complete scene by exploiting the correlation among each individual data stream from the MSNs. An AFN also serves as a relay node for other AFNs to carry traffic toward the base-station. Although an AFN is expected to be provisioned with much more energy than an MSN, it also consumes energy at a substantially higher rate (due to wireless communication over large distances). Consequently, an AFN has a limited lifetime. Upon depletion of energy at an AFN, we expect that the *coverage* for the particular area under surveillance is lost, despite the fact that some of the MSNs within the cluster may still have remaining energy.[2]

The third component in the two-tiered architecture is the base-station. The base-station is, essentially, the *sink* node for data streams from all the AFNs in the network. In this investigation, we assume that there is sufficient energy resource available at the base-station and thus there is no energy constraint at the base-station. In summary, the main functions of the lower tier MSNs are data acquisition and compression while the upper-tier AFNs are used for data fusion and relaying information to the base-station.

2.1 Power Consumption Model

For AFN i, we assume that the aggregated bit rate collected by its local MSNs *after* data fusion is g_i, $i = 1, 2, \cdots, N$. These collected local bit streams must be routed toward the base-station. Our objective is to maximize the g_i values according to the LMM criterion (see Definition 1 below) under a given network lifetime requirement.

For an AFN, energy consumption due to wireless communication (*i.e.,* receiving and transmitting) has been considered the dominant factor in power consumption [1]. The power dissipation at a radio transmitter can be modeled as:

$$p_t(i, k) = c_{ik} \cdot f_{ik} , \tag{1}$$

where $p_t(i,k)$ is the power dissipated at AFN i when it is transmitting to node k, f_{ik} is the rate transmitted from AFN i to node k, c_{ik} is the power consumption cost of radio link (i,k) and is given by

$$c_{ik} = \alpha + \beta \cdot d_{ik}^m , \tag{2}$$

where α is a *distance-independent* constant term, β is a coefficient term associated with the *distance-dependent* term, d_{ik} is the distance between these two nodes, and m is the path loss index, with $2 \leq m \leq 4$ [18]. Typical values for these parameters are $\alpha = 50$ nJ/b and $\beta = 0.0013$ pJ/b/m^4 (for $m = 4$) [10].[3] Since the power level of an AFN's transmitter can be used to control the distance coverage of an AFN (see, e.g., [16, 19, 21]), different network flow routing topologies can be formed by adjusting the power level of each AFN's transmitter.

The power dissipation at a receiver can be modeled as [18]:

$$p_r(i) = \rho \cdot \sum_{k \neq i} f_{ki} , \tag{3}$$

where $\sum_{k\neq i} f_{ki}$ (in b/s) is the rate of the received data stream at AFN i. A typical value for the parameter ρ is 50 nJ/b [10].

2.2 The LMM Rate Allocation Problem

Before we formulate the LMM rate allocation problem, let us revisit the maximum capacity problem (with "bias" in rate allocation) that was described in Section 1. For a network with N AFNs, suppose that the rate of AFN i is g_i, and that the initial energy at this node is given by e_i ($i = 1, 2, \cdots, N$). For a given network lifetime requirement T (i.e., each AFN must remain alive for at least time duration T), the maximum information capacity that the network can collect can be formulated as the following linear programming (LP).

MaxCap: Max $\sum_{i=1}^{N} g_i$
s.t.

$$f_{iB} + \sum_{k\neq i} f_{ik} - \sum_{m\neq i} f_{mi} = g_i \qquad (1 \leq i \leq N) \tag{4}$$

$$\sum_{m\neq i} \rho f_{mi} T + \sum_{k\neq i} c_{ik} f_{ik} T + c_{iB} f_{iB} T \leq e_i \qquad (1 \leq i \leq N) \tag{5}$$

$$f_{ik}, f_{iB} \geq 0 \quad (1 \leq i, k \leq N, k \neq i)$$

where f_{ik} and f_{iB} are data rates transmitted from AFN i to AFN k and from AFN i to the base-station B, respectively. The set of constraints in (4) are the flow balance equations: they state that, the total bit rate transmitted by AFN i is equal to the total bit rate received by AFN i from other AFNs, plus the bit rate generated locally at AFN i (g_i). The set of constraints in (5) are the energy constraints: they state that, for a given network lifetime requirement T, the energy required in communications (i.e., in transmitting and receiving all these data) cannot exceed the initial energy provisioning level.

Note that f_{mi}, f_{ik}, f_{iB}, and g_i are variables and that T is a constant (representing a given network lifetime requirement). MaxCap is a standard LP formulation that can be solved by a polynomial algorithm [2]. Unfortunately, as we shall see in the numerical results (Section 5), the solution to this MaxCap problem lends itself into an extreme bias toward AFNs whose data paths consume the least amount of power toward the base-station. Consequently, although the network capacity is maximized over the network lifetime T, the corresponding bit rate allocation among the AFNs (i.e., the g_i values) only favors those AFNs that have this property, while other AFNs are unfavorably allocated with much smaller (even close to 0) bit rates. As a result, the effectiveness of the network in performing information collection or surveillance could be severely compromised.

To address this fairness issue, we advocate the so-called *lexicographic max-min* (LMM) rate allocation strategy [14] in this paper, which has some similarity to the max-min rate allocation in data networks [3].[4] Under LMM rate allocation, we start with the objective of *maximizing* the bit rate for *all* the nodes until one or more nodes reach their energy constraint capacities for the given network lifetime requirement. Given that the first level of the smallest rate allocated among the nodes is maximized, we continue to

[1]Due to the small distance between an MSN and its local AFN, multi-hop routing among the MSNs may not be necessary.

[2]We assume that each MSN can only forward information to its local AFN for processing (*e.g.,* video fusion).

[3]In this paper, we use $m = 4$ in all of our numerical results.

[4]However, there is significant difference between max-min and LMM, which we will discuss shortly.

maximize the second level of rate for the remaining nodes that still have available energy, and so forth. More formally, denote $\mathbf{r} = [r_1, r_2, \cdots, r_N]$ as the sorted version (i.e., $r_1 \le r_2 \le \cdots \le r_N$) of the rate vector $\mathbf{g} = [g_1, g_2, \cdots, g_N]$, with g_i corresponding to the rate of node i. We then have the following definition for an LMM rate allocation.

DEFINITION 1. **(LMM-optimal Rate Allocation)** *For a given network lifetime requirement T, a sorted rate vector $\mathbf{r} = [r_1, r_2, \cdots, r_N]$ yields an LMM-optimal rate allocation if and only if for any other sorted rate allocation vector $\hat{r} = [\hat{r}_1, \hat{r}_2, \cdots, \hat{r}_N]$ with $\hat{r}_1 \le \hat{r}_2 \le \cdots \le \hat{r}_N$, there exists a k, $1 \le k \le N$, such that $r_i = \hat{r}_i$ for $1 \le i \le k-1$ and $r_k > \hat{r}_k$.*

Based on the LMM-optimal definition, we can calculate the first level optimal rate $\lambda_1 = r_1$ easily through the following LP.

Max λ_1
s.t.

$$f_{iB} + \sum_{k \ne i} f_{ik} - \sum_{m \ne i} f_{mi} - \lambda_1 = 0 \qquad (1 \le i \le N)$$

$$\sum_{m \ne i} \rho T f_{mi} + \sum_{k \ne i} c_{ik} T f_{ik} + c_{iB} T f_{iB} \le e_i \qquad (1 \le i \le N)$$

$$f_{ik}, f_{iB} \ge 0 \quad (1 \le i, k \le N, k \ne i)$$

Although the first level bottleneck rate λ_1 is easy to obtain, calculating the subsequent bottleneck rates are quite challenging. As discussed in Section 1, a naive approach that applies an iterative LP procedure to calculate the desired rate allocations is incorrect. This is because there is a fundamental difference in the nature of the LMM rate allocation problem described here and the classical max-min rate allocation problem in [3]. The LMM rate allocation problem implicitly *couples* a flow routing problem (i.e., a determination of the f_{ik} and f_{iB} for the entire network), while the classical max-min rate allocation explicitly assumes that the routes for all the flows are given *a priori* and fixed. Moreover, for the LMM rate allocation problem, starting from the first iteration, there usually exist *non-unique* flow routing solutions corresponding to the same maximum rate level. Consequently, each of these flow routing solutions, once chosen, will yield *different* remaining energy levels on the nodes for future iterations and so forth, leading to a different rate vector, which usually does not coincide with the LMM-optimal rate vector. Therefore, any iterative rate allocation algorithm that requires energy reservation among the nodes during each iteration is unlikely to give a correct LMM rate allocation (see Section 5 for numerical example). In the next section, we present an efficient (polynomial time) algorithm to solve the LMM rate allocation problem correctly without requiring any energy reservation during each iteration.

3. A SERIAL LP ALGORITHM BASED ON PARAMETRIC ANALYSIS

3.1 Problem Formulation

To solve the LMM rate allocation problem, we first perform the following problem formulation. Table 1 lists the notation used in this paper. Suppose that the sorted rate vector $\mathbf{r} = [r_1, r_2, \cdots, r_N]$ with $r_1 \le r_2 \le \cdots \le r_N$ is LMM-optimal. To keep track of *distinct* rates, we remove all repetitive elements in this vector and

Table 1: Notation

General Notation to the LMM-Rate and LMM-Lifetime problems	
N	The total number of AFNs in the network
e_i	The initial energy at AFN i
ρ	The power consumption coefficient for receiving data
c_{ik} (or c_{iB})	The power consumption coefficient for transmitting data from AFN i to AFN k (or the base-station B)
n	The number of distinct elements in the sorted LMM-optimal rate/lifetime vector
S_i	The minimum set of nodes that reach their energy constraint limits at i-th level
$\hat{S}_i$	The set of all possible AFNs that may reach their energy constraint limits at i-th level, $S_i \subseteq \hat{S}_i$
V_{ik} (or V_{iB})	The total volume from AFN i to AFN k (or the base-station B)
f_{ik} (or f_{iB})	The rate from AFN i to AFN k (or the base-station B)
x	The optimal solution to LMM-Rate/LMM-Lifetime
w	The optimal solution to dual problem of LMM-Rate or LMM-Lifetime
b	The right-hand-side (RHS) of LMM-Rate or LMM-Lifetime
I_i	A column vector having a single 1 element corresponding to node i in Eq. (10) or Eq. (15) and 0 for all other elements
$\mathcal{B}$	The columns corresponding to the basic variables in LMM-Rate/LMM-Lifetime
$\mathcal{Z}$	The columns corresponding to the non-basic variables in LMM-Rate/LMM-Lifetime
$c_{\mathcal{B}}$	The parameters in objective function corresponding to the basic variables of LMM-Rate/LMM-Lifetime
$c_{\mathcal{Z}}$	The parameters in objective function corresponding to the non-basic variables of LMM-Rate or LMM-Lifetime
$x_{\mathcal{B}}$	Part of optimal solution corresponding to the basic variables of LMM-Rate/LMM-Lifetime
$x_{\mathcal{Z}}$	Part of optimal solution corresponding to the non-basic variables of LMM-Rate/LMM-Lifetime
Symbols used for the LMM-Rate problem	
T	The network lifetime requirement
g_i	The local bit rate collected at AFN i
r_i	The i-th element in the sorted LMM-optimal rate vector, where $r_1 \le r_2 \le \cdots \le r_N$
λ_i	The i-th rate level in the sorted LMM-optimal rate vector, i.e., $\lambda_1 (= r_1) < \lambda_2 < \cdots < \lambda_n (= r_N)$
δ_i	$= \lambda_i - \lambda_{i-1}$, the difference between λ_i and λ_{i-1}
Symbols used for the LMM-Lifetime problem	
g_i	The rate requirement at AFN i
t_i	The node lifetime at AFN i
τ_i	The i-th element in the sorted LMM-optimal lifetime vector, where $\tau_1 \le \tau_2 \le \cdots \le \tau_N$
μ_i	The i-th drop point in the sorted LMM-optimal lifetime vector, i.e., $\mu_1 (= r_1) < \mu_2 < \cdots < \mu_n (= r_N)$
ζ_i	$= \mu_i - \mu_{i-1}$, the difference between μ_i and μ_{i-1}

rewrite it as $[\lambda_1, \lambda_2, \cdots, \lambda_n]$ such that $\lambda_1 < \lambda_2 < \cdots < \lambda_n$, where $\lambda_1 = r_1$, $\lambda_n = r_N$, and $n \le N$. For each λ_i, denote S_i, $i = 1, 2, \cdots, n$, as the corresponding set of nodes that use up their energy at this rate. Clearly, $\sum_{i=1}^{n} |S_i| = |S| = N$, where S denotes the set of all N nodes. The key to the LMM rate allocation problem is to find the correct values $\lambda_1, \lambda_2, \cdots, \lambda_n$ and the corresponding set $S_1, S_2, \cdots, S_n$, respectively.

To formulate this problem into an iterative form, we define $\lambda_0 = 0$ and $S_0 = \emptyset$. Furthermore, denote $\delta_l = \lambda_l - \lambda_{l-1}$. Starting with $l = 1$ ($1 \le l \le n$), we have an iterative optimization problem as follows.

Max δ_l
s.t.

$$f_{iB} + \sum_{k \ne i} f_{ik} - \sum_{m \ne i} f_{mi} - \delta_l = \lambda_{l-1} \qquad (i \notin \bigcup_{h=0}^{l-1} S_h) \quad (6)$$

$$f_{iB} + \sum_{k \ne i} f_{ik} - \sum_{m \ne i} f_{mi} = \lambda_h \qquad (i \in \bigcup_{h=1}^{l-1} S_h) \quad (7)$$

$$\sum_{m \ne i} \rho T f_{mi} + \sum_{k \ne i} c_{ik} T f_{ik} + c_{iB} T f_{iB} \le e_i \qquad (i \notin \bigcup_{h=0}^{l-1} S_h) \quad (8)$$

$$\sum_{m \ne i} \rho T f_{mi} + \sum_{k \ne i} c_{ik} T f_{ik} + c_{iB} T f_{iB} = e_i \qquad (i \in \bigcup_{h=1}^{l-1} S_h) \quad (9)$$

$$f_{ik}, f_{iB} \ge 0 \quad (1 \le i, k \le N, k \ne i)$$

Note that for $l = 1$, the constraints (7) and (9) do not exist. For $2 \le l \le n$, constraints (7) and (9) are for those nodes that have already reached their LMM rate allocation during the previous $l - 1$ iterations. In particular, the set of constraints in (7) say that the sum of in-coming and local data rates are equal to the out-going data rates for each node with its LMM-optimal rate λ_h, $1 \le h < l$. The set of constraints in (9) say that for these nodes that have already reached their LMM-optimal rate, the total energy consumed for communications has reached their initial energy provisioning at these nodes. On the other hand, the constraints in (6) and (8) are for the remaining nodes that have not yet reached their LMM-optimal rate. Specifically, the set of constraints in (6) state that, for these nodes that have not yet reached their energy constraint levels, the sum of in-coming and local data rates are equal to the out-going data rates. Note that the objective function is to maximize the additional rate δ_l for these nodes. Furthermore, for these nodes, the set of constrains in (8) state that the total energy consumed for communications should be upper bounded by the initial energy provisioning.

To facilitate our later discussion on duality results in Section 4, we further re-formulate above LP. In particular, we multiply both sides of (6) and (7) by T (which is a constant representing a given network lifetime requirement) and denote $V_{iB} = f_{iB}T$, $V_{ik} = f_{ik}T$, $V_{mi} = f_{mi}T$. Intuitively, V_{ik} and V_{iB} represent the bit volume that is transferred from node i to k and from node i to B, respectively, during lifetime T. We obtain the following problem formulation.

LMM-Rate: Max δ_l
s.t.

$$V_{iB} + \sum_{k \ne i} V_{ik} - \sum_{m \ne i} V_{mi} - \delta_l T = \lambda_{l-1} T \quad (i \notin \bigcup_{h=1}^{l-1} S_h) \quad (10)$$

$$V_{iB} + \sum_{k \ne i} V_{ik} - \sum_{m \ne i} V_{mi} = \lambda_h T \quad (i \in \bigcup_{h=1}^{l-1} S_h)$$

$$\sum_{m \ne i} \rho V_{mi} + \sum_{k \ne i} c_{ik} V_{ik} + c_{iB} V_{iB} \le e_i \qquad (i \notin \bigcup_{h=1}^{l-1} S_h)$$

$$\sum_{m \ne i} \rho V_{mi} + \sum_{k \ne i} c_{ik} V_{ik} + c_{iB} V_{iB} = e_i \qquad (i \in \bigcup_{h=1}^{l-1} S_h)$$

$$V_{ik}, V_{iB} \ge 0 \quad (1 \le i, k \le N, k \ne i)$$

The above LP formulation can be rewritten in the form **Max** cx, **s.t.** $Ax = b$ and $x \ge 0$, the dual problem for which is **Min** wb, **s.t.** $wA \ge c$ with w being unrestricted in sign [2]. Both can be solved by standard LP techniques (e.g., [2]). Although a solution to the LMM-Rate problem gives the optimal solution for δ_l at iteration l, it remains to determine the *minimum* set of nodes corresponding to this δ_l, which is the key difficulty in the LMM rate allocation problem. In the rest of this section, we exploit the parametric analysis technique [2] to determine the minimum node set at each rate.

3.2 Minimum Node Set Determination

Denote $\hat{S}_l$ ($\hat{S}_l \ne \emptyset$) the set of nodes for which the constraints (8) are *binding* at the l-th iteration for the LMM-Rate problem, *i.e.*, $\hat{S}_l$ include all the nodes that achieve *equality* in (8) at iteration l. Although it is certain that at least one of the nodes in $\hat{S}_l$ belong to S_l (the minimum node set for rate λ_l), for other nodes in $\hat{S}_l$, it may still be possible to further increase their rates under alternative flow routing solutions. In other words, if $|\hat{S}_l| = 1$, then we must have $S_l = \hat{S}_l$; otherwise, we must determine the *minimum* node set S_l ($S_l \subseteq \hat{S}_l$) that achieves the LMM-optimal rate allocation.

We find that the so-called *parametric analysis* (PA) technique [2] is a powerful technique to address this problem. The main idea of PA is to investigate how an infinitesimal perturbation on some components of the LMM-Rate problem can affect the objective function. In particular, considering a small increase on the right-hand-side (RHS) of (10), *i.e.*, changing b_i to $b_i + \epsilon_i$, where $\epsilon_i > 0$, node i belongs to the minimum node set S_l *if and only if* $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0) < 0$. That is, node i belongs to the minimum node set S_l if and only if a small increase in node i's rate (in terms of total volume generated at node i) leads to a *decrease* in the objective function.

To compare $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0)$ with 0, we apply an important duality results from LP theory. If x and w are the respective optimal solution to the primal and dual problems, then based on the parametric duality property [2], we have

$$\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0) = \frac{\partial^+ (cx)}{\partial b_i}(b_i) \le w_i \ . \qquad (11)$$

Recall that these w_i can be easily obtained at the same time when we solve the primal LP problem. Note that by the nature of the problem, we have $w_i \le 0$ for an optimal dual solution. Therefore, if we find that $w_i < 0$, then we can determine immediately that node i must belong to the minimum node set S_l. On the other hand, if we find that $w_i = 0$, it is not clear whether $\frac{\partial^+ \delta_l}{\partial \epsilon_i}(0)$ is strictly negative or 0 and further analysis is thus needed.

For each node i with $w_i = 0$, we must perform a complete PA to see whether a perturbation (i.e., tiny increase) on the RHS of (10) will result in any change in the objective function. If there is no change, then we can determine that node i does not belong to the minimum node set S_l; otherwise, node i belongs to S_l. Assume that the optimal solution is $(x_{\mathcal{B}}, x_{\mathcal{Z}})$, where $x_{\mathcal{B}}$ and $x_{\mathcal{Z}}$ denote the set of basic and non-basic variables; $\mathcal{B}$ and $\mathcal{Z}$ denote the columns corresponding to the basic and non-basic variables. $c_{\mathcal{B}}$ and $c_{\mathcal{Z}}$ denote the objective function coefficient vectors for the basic and non-basic variables; and q denotes the objective value. Then we have the corresponding canonical equations as follows

$$\begin{aligned} q + (c_{\mathcal{B}}^t \mathcal{B}^{-1}\mathcal{Z} - c_{\mathcal{Z}}^t)x_{\mathcal{Z}} &= c_{\mathcal{B}}^t \mathcal{B}^{-1} b \ , \\ x_{\mathcal{B}} + (\mathcal{B}^{-1}\mathcal{Z})x_{\mathcal{Z}} &= \mathcal{B}^{-1} b \ . \end{aligned}$$

If b is replaced by $b + \epsilon_i I_i$, where the column vector I_i has a single 1 element corresponding to node i in the set of constraints (10) while all the other elements are 0, then the only change due to this perturbation is that $\mathcal{B}^{-1}b$ will be replaced by $\mathcal{B}^{-1}(b + \epsilon_i I_i)$. Consequently, the objective value for the current basis becomes $c_{\mathcal{B}}^t \mathcal{B}^{-1}(b + \epsilon_i I_i)$. As long as $\mathcal{B}^{-1}(b + \epsilon_i I_i)$ is nonnegative, the current basis remains optimal. Denote $\bar{b} = \mathcal{B}^{-1}b$, $\mathcal{B}_i^{-1} = \mathcal{B}^{-1}I_i$, and let $\hat{\epsilon}_i$ be an upper bound for ϵ_i such that the current basis remains optimal. We have

$$\hat{\epsilon}_i = \min_j \left\{ \frac{\bar{b}_j}{-\mathcal{B}_{ij}^{-1}} : \mathcal{B}_{ij}^{-1} < 0 \right\} \ . \tag{12}$$

If $\hat{\epsilon}_i > 0$, the optimal objective value varies according to $c_{\mathcal{B}}^t \mathcal{B}^{-1}(b + \epsilon_i I_i)$ for $0 < \epsilon_i \le \hat{\epsilon}_i$. Since $w = c_{\mathcal{B}}^t \mathcal{B}^{-1}$ and $w_i = 0$, we have $c_{\mathcal{B}}^t \mathcal{B}^{-1} I_i = w_i = 0$. Thus, the objective value will *not* change for $\epsilon_i \in (0, \hat{\epsilon}_i]$, and consequently, the rate for node i can be increased beyond the current λ_l value. That is, node i does not belong to the minimum node set S_l.

For most problems in practice, the above procedure is sufficient to determine whether or not node i belongs to the minimum node set S_l for all $i \in \hat{S}_l$. But in the rare event where $\hat{\epsilon}_i = 0$, the problem is degenerate. To develop a polynomial-time algorithm, denote W_l as the set of all nodes with $w_i < 0$ and U_l as the set of all nodes with $w_i = 0$ and $\hat{\epsilon}_i = 0$. Then we solve the following LP to maximize the slack variables (SV) for nodes in U_l.

MSV: Max $\sum_{i \in U_l} \epsilon_i$
s.t.

$$\begin{aligned} V_{iB} + \sum_{k \ne i} V_{ik} - \sum_{m \ne i} V_{mi} - \epsilon_i T = \lambda_l T \qquad & (i \in U_l) \\ V_{iB} + \sum_{k \ne i} V_{ik} - \sum_{m \ne i} V_{mi} = \lambda_h T \qquad & (i \in \textstyle\bigcup_{h=1}^{l-1} S_h) \\ V_{iB} + \sum_{k \ne i} V_{ik} - \sum_{m \ne i} V_{mi} = \lambda_l T \qquad & (i \notin U_l \textstyle\bigcup_{h=1}^{l-1} S_h) \\ \sum_{m \ne i} \rho V_{mi} + \sum_{k \ne i} c_{ik} V_{ik} + c_{iB} V_{iB} = e_i \quad & (i \in U_l \textstyle\bigcup W_l \bigcup_{h=1}^{l-1} S_h) \\ \sum_{m \ne i} \rho V_{mi} + \sum_{k \ne i} c_{ik} V_{ik} + c_{iB} V_{iB} \le e_i \quad & (i \notin U_l \textstyle\bigcup W_l \bigcup_{h=1}^{l-1} S_h) \\ V_{ik}, V_{iB}, \epsilon_i \ge 0 \quad & (1 \le i, k \le N, k \ne i) \end{aligned}$$

If the optimal objective function is 0, then we conclude that no node in U_l can have a positive ϵ_i. That is, these nodes should all belong to S_l and we have $S_l = W_l + U_l$. On the other hand, if the optimal objective function is positive, then some nodes $i \in U_l$ must have positive ϵ_i values and these nodes therefore do not belong to the minimum node set S_l. Consequently, we can remove these nodes from U_l. If $U_l \ne \emptyset$, we move on to solve another MSV. This procedure will terminate when the optimal objective function value is 0 or $U_l = \emptyset$.

To ensure that MSV determinate the minimum node set correctly, we need the following lemma. The proof is give in [11].

LEMMA 1. **(The Minimum Node Set is Unique.)** *The minimum node set for each rate level under the LMM-optimal rate allocation is unique.*

In a nutshell, the complete PA procedure to determine whether a node $i \in \hat{S}_l$ belongs to the minimum node set S_l can be summarized as follows.

ALGORITHM 1. **(Minimum Node Set Determination with PA)**

1. *Initialize sets* $W_l = \emptyset$ *and* $U_l = \emptyset$.
2. *For each node* $i \in \hat{S}_l$,
   (a) *If* $w_i < 0$, *then* $W_l = W_l \cup \{i\}$.
   (b) *Otherwise (i.e.,* $w_i = 0$*), compute* $\bar{b} = \mathcal{B}^{-1}b$, $\mathcal{B}_i^{-1} = \mathcal{B}^{-1}I_i$, *and* $\hat{\epsilon}_i$ *according to (12).*
   If $\hat{\epsilon}_i = 0$, *then* $U_l = U_l + \{i\}$.
3. *If* $U_l = \emptyset$, *then* $S_l = W_l$ *and stop;*
   else set up the MSV problem and solve it.
4. *If the optimal objective value in MSV is* 0, *then* $S_l = W_l + U_l$ *and stop; else remove all nodes* i *with* $\epsilon_i > 0$ *from the set* U_l *and go to Step 3.*

3.3 Optimal Flow Routing for LMM Rate Allocation

Once we solve the LMM rate allocation problem, the corresponding optimal flow routing can be easily obtained by dividing the total bit volume on each link (V_{ik} or V_{iB}) by T, i.e.,

$$f_{ik} = \frac{V_{ik}}{T} \ , \tag{13}$$

$$f_{iB} = \frac{V_{iB}}{T} \ , \tag{14}$$

where T is the given network lifetime requirement. Although the LMM-optimal rate allocation is unique, it is important to note that the corresponding optimal flow routing solution is *not* unique. This is because upon the completion of the LMM rate allocation problem (i.e., upon finding $[\lambda_1, \lambda_2, \cdots, \lambda_n]$), there usually exist non-unique bit volume solutions (V_{ik} and V_{iB} values) corresponding to the same LMM-optimal rate allocation. This result is summarized in the following lemma.

LEMMA 2. *The optimal flow routing solution corresponding to the LMM rate allocation may not be unique.*

3.4 Complexity Analysis

We now analyze the complexity of the SLP-PA algorithm in solving the LMM rate allocation problem. First we consider the complexity of finding each node's rate and the total bit volume transmitted along each link. At each stage, we solve an LP problem, both its primal and dual have a complexity of $O({n_A}^3 L)$ [2], where n_A is the number of constraints or variables in the problem, whichever is larger, and L is the number of binary bits required to store the data. Since the number of variables is $O(N^2)$ and is larger than the number of constraints (which is $O(N)$), the complexity of solving the LP is $O(N^6 L)$. After solving an LP at each stage, we need to determine whether or not a node that just reached its energy binding constraint belongs to the minimum node set for this stage. Note that w and $\hat{b} = \mathcal{B}^{-1}b$ can be readily obtained when we solve the primal LP problem. To determine whether a node, say i, belongs to the minimum node set, we examine w_i. If $w_i < 0$, then node i belongs to the minimum node set and the complexity is $O(1)$. On the other hand, if $w_i = 0$, we need to further examine whether $\hat{\epsilon}_i > 0$ or not. Based on (12), the computation for $\hat{\epsilon}_i$ is $O(N)$. So at each stage, the complexity in PA for each node is $O(N)$. The total complexity of PA at each stage for the node set is thus $|\hat{S}_l| \cdot O(N)$ or $O(N \cdot N) = O(N^2)$. Thus, the complexity at each stage is $O(N^6 L) + O(N^2) = O(N^6 L)$. As there are at most N stages, the overall complexity is $O(N^7 L)$.

We now analyze the complexity for the degenerate case. Upon the completion of Step 2 in Algorithm 1, we denote $U_l^{(0)} = U_l$. Since we need to solve at most $|U_l^{(0)} - S_l|$ LPs, the complexity is $|U_l^{(0)} - S_l| \cdot O(N^6 L)$ or $O(N \cdot N^6 L) = O(N^7 L)$. Hence, the complexity at each stage is $O(N^6 L) + O(N^2) + O(N^7 L) = O(N^7 L)$. Since there are at most N stages, the overall complexity is $O(N^8 L)$.

The complexity in finding the optimal flow routing is bounded by the number of radio links in the network, which is $O(N^2)$. Hence the overall complexity is $O(N^7 L) + O(N^2) = O(N^7 L)$ for the non-degenerate case and $O(N^8 L) + O(N^2) = O(N^8 L)$ for the degenerate case. Under either case, the computational complexity is polynomial.[5]

4. EXTENSION TO LMM NODE LIFETIME PROBLEM AND DUALITY THEOREM

In this section, we present two important extensions for our results in Section 3. First, we show that our SLP-PA algorithm can be used to solve the maximum node lifetime curve problem in [6], which we define as the LMM node lifetime problem. We show that the SLP-PA algorithm is a much more efficient approach than the one proposed in [6]. Second, we show that there exists an elegant duality relationship between the LMM rate allocation problem and the LMM node lifetime problem. Consequently, important results and insights can be drawn by simply solving one of the two problems.

4.1 The LMM-optimal Node Lifetime Problem and Solution

The LMM node lifetime problem considers the following scenario. For a network with N AFNs, with a given local bit rate g_i (fixed) and initial energy e_i for AFN i, $i = 1, 2, \cdots, N$, how can we maximize the network lifetime for *all* AFNs in the network? In other words, the LMM node lifetime problem not only considers how to maximize the network lifetime until the first AFN runs out of energy, but also the time for all the AFNs in the network.

More formally, for each AFN i, denote the corresponding lifetime as t_i, $i = 1, 2, \cdots, N$. Note that g_i are fixed here, while t_i are the optimization variables, which are different from the LMM rate allocation problem that we studied in the last section. Denote $[\tau_1, \tau_2, \cdots, \tau_N]$ as the *sorted* sequence of the t_i values in nondecreasing order. Then LMM-optimal node lifetime can be defined as follows.

DEFINITION 2. **(LMM-optimal Node Lifetime)** *A sorted node lifetime vector $[\tau_1, \tau_2, \cdots, \tau_N]$ with $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_N$ is LMM-optimal if and only if for any other sorted node lifetime vector $[\hat{\tau}_1, \hat{\tau}_2, \cdots, \hat{\tau}_N]$ with $\hat{\tau}_1 \leq \hat{\tau}_2 \leq \cdots \leq \hat{\tau}_N$, there exists a k, $1 \leq k \leq N$, such that $\tau_i = \hat{\tau}_i$ for $1 \leq i \leq k-1$ and $\tau_k > \hat{\tau}_k$.*

[5] Note that our analysis here give a loose upper bound for time complexity. In practice, the actual running time for LP implementation is much faster than its upper bound.

Solution. It should be clear that, under the LMM-optimal node lifetime objective, we must *maximize* the time until a set of nodes use up their energy (which is also called a *drop point* in [6]) while *minimizing* the number of nodes that drain up their energy at each drop point. We now show that the SLP-PA algorithm developed for the LMM rate allocation problem can be directly applied to solve the LMM node lifetime problem.

Suppose that $[\tau_1, \tau_2, \cdots, \tau_N]$ with $\tau_1 \leq \tau_2 \leq \cdots \leq \tau_N$ is LMM-optimal. To keep track of *distinct* node lifetimes (or drop points) in this vector, we remove all repetitive elements in the vector and rewrite it as $[\mu_1, \mu_2, \cdots, \mu_n]$ such that $\mu_1 < \mu_2 < \cdots < \mu_n$, where $\mu_1 = \tau_1$, $\mu_n = \tau_N$, and $n \leq N$. Corresponding to these drop points, denote $S_1, S_2, \cdots, S_n$ as the sets of nodes that drain up their energy at drop points $\mu_1, \mu_2, \cdots, \mu_n$, respectively. Then $|S_1| + |S_2| + \cdots + |S_n| = |S| = N$, where S denotes the set of all N AFNs in the network. The problem is to find the LMM-optimal values of $\mu_1, \mu_2, \cdots, \mu_n$ and the corresponding sets $S_1, S_2, \cdots, S_n$.

Similar to the LMM rate allocation problem, the LMM node lifetime problem can be formulated as an iterative optimization problem as follows. Denote $\mu_0 = 0$, $S_0 = \emptyset$, and $\zeta_l = \mu_l - \mu_{l-1}$. Starting from $l = 1$, we solve the following LP iteratively.

LMM-Lifetime: Max ζ_l

s.t.

$$V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} - \zeta_l g_i = \mu_{l-1} g_i \qquad (i \notin \textstyle\bigcup_{h=0}^{l-1} S_h) \quad (15)$$

$$V_{iB} + \sum_{k \neq i} V_{ik} - \sum_{m \neq i} V_{mi} = \mu_h g_i \qquad (i \in \textstyle\bigcup_{h=0}^{l-1} S_h)$$

$$\sum_{m \neq i} \rho V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} \leq e_i \qquad (i \notin \textstyle\bigcup_{h=0}^{l-1} S_h)$$

$$\sum_{m \neq i} \rho V_{mi} + \sum_{k \neq i} c_{ik} V_{ik} + c_{iB} V_{iB} = e_i \qquad (i \in \textstyle\bigcup_{h=0}^{l-1} S_h)$$

$$V_{ik}, V_{iB}, \zeta_l \geq 0 \quad (1 \leq i, k \leq N, k \neq i)$$

Comparing the LMM-Lifetime problem here to the LMM-Rate problem that we studied in Section 3.1, we find that they are exactly of the same form. The only differences are that under the LMM-Lifetime problem, the local bit rates g_i are constants and the node lifetimes τ_i are variables (subject to optimization), while under the LMM-Rate problem, the g_i are variables (subject to optimization) and the node lifetimes are all identical (T), $i = 1, 2, \cdots, N$. Since the mathematical formulation for the two problems are identical, we can apply the SLP-PA algorithm to solve the LMM node lifetime problem as well.

The only issue that we need to be concerned about is the optimal flow routing solution corresponding to the LMM-optimal lifetime vector. The optimal flow routing solution here is not as simple as that for the LMM rate allocation problem, which merely involves a simple division (see Eqs. (13) and (14)). We refer readers to [11] for an $O(N^4)$ algorithm to obtain an optimal flow routing solution for the LMM-optimal lifetime vector. Similar to Lemma 2, the optimal flow routing solution corresponding to the LMM node lifetime problem may not be unique.

Complexity Comparison. In [6], Brown *et al.* studied the LMM node lifetime problem under the so-called "maximum node lifetime curve" problem. They also made an important contribution by developing the first procedure to solve this problem correctly. A key step in their procedure is the use of *multiple* independent LP calculations to determine the minimum node set at each drop point, which we call *serial LP with slack variable analysis* (SLP-SV). Although this approach solves the LMM node lifetime problem correctly, its computational complexity (potentially exponential) remains an issue to be resolved.

On the other hand, the SLP-PA algorithm developed in this paper is strictly polynomial and is computationally more efficient than the SLP-SV approach. To understand the difference between the two, we take a closer look on the computational complexity of the SLP-SV approach in [6]. First, SLP-SV needs to keep track of each *sub-flow* along its route from the source node toward the base-station. Such a flow-based (or more precisely, sub-flow based) approach could make the size of the LP coefficient matrix exponential, which leads to an exponential-time algorithm [2].[6]

Second, even if a link-based LP formulation such as ours is adopted in [6], the computational efficiency of the SV-based approach is still worse than the SLP-PA algorithm. This is because at each stage, the SV-based approach must solve several *additional* LPs (up to $|\hat{S}_l - S_l|$) to determine S_l, which is in contrast to the simpler PA under the SLP-PA algorithm ($O(N^2)$). Even for the degenerate case, the number of additional LPs under the SLP-PA algorithm is at most $|U_l^{(0)} - S_l|$,[7] which is still no more than $|\hat{S}_l - S_l|$.

Finally, we discuss a hybrid link-flow approach mentioned in [6]. In this approach, link-based formulations are used for sub-flows. This leads to a much fewer number of variables than those for the flow-based approach. But this approach still requires sub-flow accounting and results in an order of magnitude more constraints than the link-based approach in SLP-PA. Although this approach solves the LMM node lifetime problem in polynomial-time (*e.g.,* by using interior point methods [2]), the overall complexity is still orders of magnitude higher than that under the SLP-PA algorithm. Furthermore, the burden of solving additional LPs to determine whether a node belongs to the minimum node set still remains.

4.2 Duality Theorem

In this section, we present an elegant and powerful result showing that there is an underlying duality relationship between the LMM rate allocation problem and the LMM node lifetime problem. Consequently, the solutions and insights obtained for one problem can be "mirrored" to the other problem.

To start with, we denote $\mathcal{P}_R$ as the LMM rate allocation problem where we have N AFNs in the network and all nodes have a common given lifetime requirement T (constant). Denote g_i as the LMM-optimal rate allocation for node i under $\mathcal{P}_R$, $i = 1, 2, \cdots, N$. Similarly, we denote $\mathcal{P}_L$ as the LMM node lifetime problem where all nodes have the same local bit rate R (constant). Denote t_i as the LMM node lifetime for node i under $\mathcal{P}_L$, $i = 1, 2, \cdots, N$. Then the following theorem shows how the solution to one problem can be used to obtain the solution to the other.

[6]Incidentally, the revised simplex method proposed in [6] is not as efficient as the polynomial-time algorithm described in [2] and is itself exponential.

[7]Recall that $U_l^{(0)}$ denotes U_l upon the completion of Step 2 in Algorithm 1.

Table 2: Duality relationship between LMM rate allocation problem $\mathcal{P}_R$ and LMM node lifetime problem $\mathcal{P}_L$.

LMM rate allocation ($\mathcal{P}_R$)	LMM node lifetime ($\mathcal{P}_L$)
g_i (optimization variable)	$g_i = R$ (constant)
$t_i = T$ (constant)	t_i (optimization variable)
Total bit volume at AFN i: $g_i \cdot T = t_i \cdot R$	

THEOREM 1. **(Duality Theorem)**
For a given node lifetime requirement T for all nodes under problem $\mathcal{P}_R$ and a given local bit rate R for all nodes under problem $\mathcal{P}_L$, we have the following relationship between the solutions to the LMM rate allocation problem $\mathcal{P}_R$ and the LMM node lifetime problem $\mathcal{P}_L$.
(i) Suppose that we have solved problem $\mathcal{P}_R$ and obtained the LMM-optimal rate allocation g_i for each node i ($i = 1, 2, \cdots, N$). Then under $\mathcal{P}_L$, the LMM node lifetime t_i for node i is

$$t_i = \frac{g_i T}{R} \ . \tag{16}$$

(ii) Suppose that we have solved problem $\mathcal{P}_L$ and obtained the LMM-optimal node lifetime t_i for each node i ($i = 1, 2, \cdots, N$). Then under $\mathcal{P}_R$, the LMM rate allocation g_i for node i is

$$g_i = \frac{t_i R}{T} \ . \tag{17}$$

Table 2 shows the duality relationship between solutions to problems $\mathcal{P}_R$ and $\mathcal{P}_L$.

Proof. We prove (i) and (ii) in Theorem 1 separately.
(i) We organize our proof into two parts. First, we show that t_i are feasible node lifetimes in terms of flow balance and energy constraints on each node i ($i = 1, 2, \cdots, N$). Then we show that it is indeed the LMM-optimal node lifetime.

Feasibility. Since we have obtained the solution to problem $\mathcal{P}_R$, we have one feasible flow routing solution for sending bit streams g_i, $i = 1, 2, \cdots, N$, to the base-station. Under problem $\mathcal{P}_R$, the bit volumes (V_{ij} and V_{iB} values) must meet the following equalities under the LMM-optimal rate allocation:

$$V_{iB} + \sum_{1 \le k \le N, k \ne i} V_{ik} - \sum_{1 \le m \le N, m \ne i} V_{mi} = g_i T \ ,$$
$$\sum_{1 \le m \le N, m \ne i} \rho V_{mi} + \sum_{1 \le k \le N, k \ne i} c_{ik} V_{ik} + c_{iB} V_{iB} = e_i \ .$$

Now replacing $g_i T$ by $t_i R$, we see that the *same* bit volume solution under $\mathcal{P}_R$ yields a feasible bit volume solution to the node lifetime problem under $\mathcal{P}_L$. Consequently, we can obtain the flow routing solution to problem $\mathcal{P}_L$ under the bit volume solution to problem $\mathcal{P}_L$ [11] and this verifies that t_i, $i = 1, 2, \cdots, N$, is a feasible solution to problem $\mathcal{P}_L$.

Optimality. To prove that the t_i, $i = 1, 2, \cdots, N$, obtained via (16) are indeed LMM-optimal for problem $\mathcal{P}_L$, we sort g_i, $i = 1, 2, \cdots, N$, under problem $\mathcal{P}_R$ in non-decreasing order and denote it as $[r_1, r_2, \cdots, r_N]$. We also introduce a node index $I = [i_1, i_2, \cdots, i_N]$ for $[r_1, r_2, \cdots, r_N]$. For example, $i_3 = 7$ means that r_3 actually corresponds to the rate of AFN 7, i.e., $r_3 = g_7$.

Since t_i is proportional to g_i through the relationship ($t_i = \frac{T}{R} \cdot g_i$), listing t_i, $i = 1, 2, \cdots, N$, according to $I = [i_1, i_2, \cdots, i_N]$

Table 3: Node coordinates for the 10-AFN network.

i	(x_i, y_i) (in meters)	i	(x_i, y_i) (in meters)
1	(400, -320)	6	(-500, 100)
2	(300, 440)	7	(-400, 0)
3	(-300, -420)	8	(420, 120)
4	(320, -100)	9	(200, 140)
5	(-120, 340)	10	(220, -340)

will also yield a *sorted* (in *non-decreasing* order) lifetime list, denoted as $[\tau_1, \tau_2, \cdots, \tau_N]$. We now prove that $[\tau_1, \tau_2, \cdots, \tau_N]$ is indeed LMM-optimal for problem $\mathcal{P}_L$.

Our proof is based on contradiction. Suppose that $[\tau_1, \tau_2, \cdots, \tau_N]$ is not LMM-optimal for problem $\mathcal{P}_L$. Assume that the LMM-optimal lifetime vector to problem $\mathcal{P}_L$ is $[\hat{\tau}_1, \hat{\tau}_2, \cdots, \hat{\tau}_N]$ (sorted in non-decreasing order) with the corresponding node index being $\hat{I} = [\hat{i}_1, \hat{i}_2, \cdots, \hat{i}_N]$. Then, by Definition 2, there exists a k such that $\hat{\tau}_j = \tau_j$ for $1 \leq j \leq k-1$ and $\hat{\tau}_k > \tau_k$.

We now claim that if $\hat{t}_i$, $i = 1, 2, \cdots, N$, is a feasible solution to problem $\mathcal{P}_L$, then $\hat{g}_i$ obtained via $\hat{g}_i = \frac{\hat{t}_i R}{T}$, $i = 1, 2, \cdots, N$, is also a feasible solution to problem $\mathcal{P}_R$. The proof for this claim follows identically as above. Using this result, we can obtain a corresponding feasible solution $[\hat{r}_1, \hat{r}_2, \cdots, \hat{r}_N]$ with $\hat{r}_i = \frac{\hat{\tau}_i R}{T}$ and the node index $\hat{I}$ for problem $\mathcal{P}_R$. Hence we have $\hat{r}_j = \frac{\hat{\tau}_j R}{T} = \frac{\tau_j R}{T} = r_j$ for $1 \leq j \leq k-1$ but $\hat{r}_k = \frac{\hat{\tau}_k R}{T} > \frac{\tau_k R}{T} = r_k$. That is, $[r_1, r_2, \cdots, r_N]$ is not LMM-optimal and this leads to a contradiction.

(ii) The proof for this part is similar to the above proof for (i) and is thus omitted here. □

This duality relationship can offer important insights on system performance issues, in addition to providing solutions to the LMM rate allocation and the LMM node lifetime problems. For example, in Section 1, we pointed out the potential bias (fairness) issue associated with the network capacity maximization objective (i.e., sum of rates from all nodes). It is interesting to see that there is a dual fairness issue under the node lifetime problem. In particular, the objective of maximizing the *sum* of node lifetimes among all nodes also leads to a bias (or fairness) problem because this objective would only favor those nodes that consume energy at a small rate. As a result, certain nodes will have much larger lifetimes while some other nodes will be penalized with much smaller lifetimes.

5. NUMERICAL INVESTIGATION

In this section, we use numerical results to illustrate our SLP-PA algorithm to the LMM rate allocation problem and compare it with other approaches. We also use numerical results to illustrate the duality relationship between the LMM rate allocation problem and the LMM node lifetime problem.

Due to space limitation, we will show results for a network of 10 AFNs. More results for network of larger size are available in [11]. In this 10-AFN network, the base-station B is located at the origin while the locations for the 10 AFNs are randomly generated over a 1000m x 1000m square area (see Fig. 2 and Table 3).

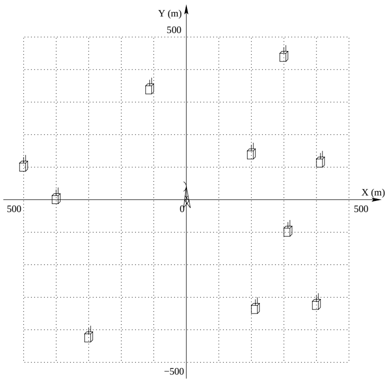


Figure 2: Network topology used in the numerical investigation.

Table 4: Rate allocation under the SLP-PA, SLP, and MaxCap approaches for the 10-AFN Network.

i (Sorted Node Index)	SLP-PA		SLP		MaxCap	
	r_i (kb/s)	AFN	r_i (kb/s)	AFN	r_i (kb/s)	AFN
1	0.1023	3	0.1023	1	0.0553	2
2	0.1023	6	0.1023	2	0.0627	3
3	0.1023	7	0.1023	3	0.0646	1
4	0.1536	5	0.1023	6	0.0658	6
5	0.2941	1	0.1023	7	0.1222	8
6	0.2941	2	0.1536	5	0.1653	10
7	0.2941	4	0.1536	8	0.1736	7
8	0.2941	8	0.1536	10	0.2628	5
9	0.2941	9	0.6563	4	0.3513	4
10	0.2941	10	0.6563	9	1.2398	9

5.1 SLP-PA Algorithm to the LMM Rate Allocation Problem

We will compare SLP-PA with the naive approach (see Section 2.2) that uses a serial LP "blindly" to solve the LMM rate allocation problem. We call this naive approach *Serial LP* (SLP). As discussed in Section 2.2, the naive SLP approach requires energy reservation at each stage and will not give the correct final solution to the LMM rate allocation problem.

We will also compare our SLP-PA algorithm with the *Maximum-Capacity* (MaxCap) approach (see Section 2.2). As discussed in the beginning of Section 2.2, the rate allocation under the MaxCap approach can be extremely biased and favors only those AFNs that consume the least power along their data paths toward the base-station.

We assume that the initial energy at each AFN is 50 kJ and that under the LMM rate allocation problem, the network lifetime requirement is 100 days. The power consumption behaviors for transmission and reception are defined in (1) and (3), respectively.

Table 4 shows the rate allocation for the AFNs under each approach, which is also plotted in Fig. 3. The "sorted node index" corresponds the sorted rates among the AFNs in non-decreasing order.

Clearly, among the three rate allocation approaches, only the rate allocation under SLP-PA meets the LMM-optimal rate allocation definition (see Definition 1) when compared with the rate allocation under SLP and MaxCap. Specifically, comparing SLP-

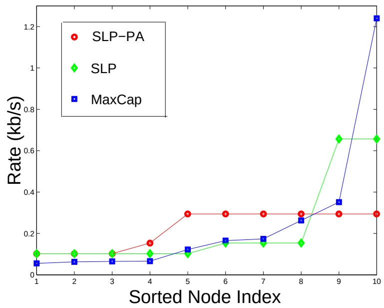


Figure 3: Rate allocation under the SLP-PA, SLP, and MaxCap approaches for a 10-AFN network .

PA with SLP, we have $r_1^{\text{SLP-PA}} = r_1^{\text{SLP}}$, $r_2^{\text{SLP-PA}} = r_2^{\text{SLP}}$, $r_3^{\text{SLP-PA}} = r_3^{\text{SLP}}$, and $r_4^{\text{SLP-PA}} > r_4^{\text{SLP}}$; comparing SLP-PA with MaxCap, we have $r_1^{\text{SLP-PA}} > r_1^{\text{MaxCap}}$.

We also observe, as expected, a severe bias in the rate allocation under the MaxCap approach. In particular, r_{10} alone accounts for over 48% of the sum of total rates among all the AFNs. Comparing the three approaches, we have $r_1^{\text{SLP-PA}} = r_1^{\text{SLP}} > r_1^{\text{MaxCap}}$ and $r_{10}^{\text{SLP-PA}} < r_{10}^{\text{SLP}} < r_{10}^{\text{MaxCap}}$. In other words, the rate allocation vector under the SLP-PA algorithm has the smallest rate difference between the smallest rate (r_1) and the largest rate (r_{10}), i.e., $r_{10} - r_1$, among the three approaches. In addition, although $r_1^{\text{SLP-PA}} = r_1^{\text{SLP}}$ for the first level rate allocation, the minimum node set for $r_1^{\text{SLP-PA}}$ is smaller than the minimum node set for r_1^{SLP}, i.e., $|S_1^{\text{SLP-PA}}| = 3 < |S_1^{\text{SLP}}| = 5$. This confirms that the naive SLP approach cannot offer the correct solution to the LMM rate allocation problem.

5.2 Duality Results

We now use numerical results to verify the duality relationship between the LMM rate allocation problem ($\mathcal{P}_R$) and the LMM node lifetime problem ($\mathcal{P}_L$) (see Section 4.2). Again, we use the 10-AFN network configurations in Fig. 2. The coordinates for each AFN under the 10-AFN network are listed in Table 3. We assume that the initial energy at each AFN is 50 kJ and that the network lifetime requirement under the LMM rate allocation problem is $T = 100$ days. Under $\mathcal{P}_L$, we assume the local bit rate for all AFNs are $R = 0.2$ kb/s.

To verify the duality relationship (Theorem 1), we perform the following calculations. First, we solve the LMM rate allocation problem ($\mathcal{P}_R$) and the LMM node lifetime problem ($\mathcal{P}_L$) *independently* with the above initial conditions using the SLP-PA algorithm. Consequently, we obtain the LMM-optimal rate allocation (g_i for each AFN i) under $\mathcal{P}_R$ and the LMM-optimal node lifetime (t_i for each AFN i) under $\mathcal{P}_L$. Then we compute $T \cdot g_i$ and $R \cdot t_i$ separately for each AFN i and examine if they are equal to each other.

The results for the LMM-optimal rate allocation ($g_i, i = 1, 2, \cdots, 10$) and the LMM-optimal node lifetime ($t_i, i = 1, 2, \cdots, 10$) for the 10-AFN network are shown in Table 5. We find that $T \cdot g_i$ and $R \cdot t_i$ are exactly equal for all AFNs, precisely as we would expect under Theorem 1.

Table 5: Numerical results verifying the duality relationship $T \cdot g_i = R \cdot t_i$ between the LMM rate allocation problem ($\mathcal{P}_R$) and the LMM node lifetime problem ($\mathcal{P}_L$) for the 10-AFN network.

AFN	$\mathcal{P}_R$ ($T = 100$ days)		$\mathcal{P}_L$ ($R = 0.2$ kb/s)	
	g_i	$T \cdot g_i$	t_i	$R \cdot t_i$
1	0.2941	29.41	147.07	29.41
2	0.2941	29.41	147.07	29.41
3	0.1023	10.23	51.17	10.23
4	0.2941	29.41	147.07	29.41
5	0.1536	15.36	76.79	15.36
6	0.1023	10.23	51.17	10.23
7	0.1023	10.23	51.17	10.23
8	0.2941	29.41	147.07	29.41
9	0.2941	29.41	147.07	29.41
10	0.2941	29.41	147.07	29.41

6. RELATED WORK

Due to energy constraints in wireless sensor networks, there has been active research on exploring the performance limits of such networks. These performance limits include, among others, *network capacity* and *network lifetime*. Network capacity typically refers to the maximum amount of bit volume that can be successfully delivered to the base-station ("sink node") by all the nodes in the network, where network lifetime refers to the maximum time that the nodes in the network remain alive before one or more nodes deplete their energy.

The network capacity problem and network lifetime problem have so far been studied disjointly in the literature. For example, in [12], the problem of how to maximize network capacity via routing was studied. While, in many other efforts (see, e.g., [4, 5, 8, 13, 22]), the focus was on how to maximize the time until the first node drains up its energy.

In this paper, we study the important overarching problem that considers both network capacity and network lifetime. Under the LMM rate allocation problem, we studied how to maximize rate allocations for *all* the nodes in the network under a given network lifetime requirement. Under the LMM node lifetime problem, we studied how to maximize the lifetime for *all* nodes when the local bit rate for each node is given *a priori*. The LMM rate allocation criterion effectively mitigates the unfairness issue when the objective is to maximize the total bit volume generated by the network. Although the LMM rate allocation is somewhat similar to the classical max-min strategy [3], there is a fundamental difference between the two. In particular, the LMM rate allocation problem implicitly embeds (or couples) a flow routing problem within rate allocation, while under the classical max-min rate allocation, there is no routing problem involved since the routes for all flows are fixed. Due to this coupling of flow routing and rate allocation, a solution approach (i.e., SLP-PA) to the LMM rate allocation problem is much more challenging than that for the classical max-min.

In [20], Srinivasan *et al.* applied game theory and Nash equilibrium among the nodes to forward packets such that the total throughput (capacity) can achieve an optimal operating point subject to a common lifetime requirement on all nodes. However, the fairness issue in information collection was not considered. The most relevant work to ours is by Brown *et al.* [6], which has been discussed in detail in Section 4.1.

7. CONCLUSIONS

In this paper, we investigated the important problem of rate allocation for wireless sensor networks under a given network lifetime requirement. Since the objective of maximizing the sum of rates of all nodes can lead to a severe bias in rate allocation among the nodes, we advocate the use of *lexicographical max-min* (LMM) rate allocation for all nodes in the network. To calculate the LMM-optimal rate vector, we developed a polynomial-time algorithm by exploiting the *parametric analysis* (PA) technique from linear programming (LP), which we called *serial LP with Parametric Analysis* (SLP-PA). Furthermore, we showed that the SLP-PA algorithm can also be employed to address the maximum node lifetime curve problem and that the SLP-PA algorithm is much more efficient than existing techniques. More importantly, we discovered a simply and elegant duality relationship between the LMM rate allocation problem and LMM node lifetime problem, which enable us to develop solutions and insights on both problems by solving any one of the two problems. Our results in this paper offer some important understanding on network capacity and network lifetime problems for energy-constrained wireless sensor networks.

Our efforts in this paper have been centered on developing centralized theory on the LMM rate allocation problem and its relationship to the LMM node lifetime problem. This understanding is essential to further research on distributed implementations, which are currently underway and the results of which will be reported in a separate paper.

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