

Crossing Over the Bounded Domain: From Exponential To Power-law Inter-meeting Time in MANET

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ABSTRACT

Inter-meeting time between mobile nodes is one of the key metrics in a Mobile Ad-hoc Network (MANET) and central to the end-to-end delay and forwarding algorithms. It is typically assumed to be exponentially distributed in many performance studies of MANET or numerically shown to be exponentially distributed under most existing mobility models in the literature. However, recent empirical results show otherwise: the inter-meeting time distribution in fact follows a power-law. This outright discrepancy potentially undermines our understanding of the performance tradeoffs in MANET obtained under the exponential distribution of the inter-meeting time, and thus calls for further study on the power-law inter-meeting time including its fundamental cause, mobility modeling, and its effect. In this paper, we rigorously prove that a *finite domain*, on which most of the current mobility models are defined, plays an important role in creating the exponential tail of the inter-meeting time. We also prove that by simply removing the boundary in a simple two-dimensional isotropic random walk model, we are able to obtain the empirically observed power-law decay of the inter-meeting time. We then discuss the relationship between the size of the boundary and the relevant timescale of the network scenario under consideration. Our results thus provide guidelines on the design of new mobility models with power-law inter-meeting time distribution, new protocols including packet forwarding algorithms, as well as their performance analysis.

Categories and Subject Descriptors: C.2.1 [Computer-Communication Networks]: Network Architecture and Design - *Wireless communication*

General Terms: Theory

Keywords: mobile ad-hoc network, inter-meeting time distribution, exponential vs. power-law, bounded domain, time and space scaling

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1. INTRODUCTION

Capacity and delay are two main factors in assessing the performance of MANETs. In order to successfully transfer data from a mobile node to another, the mobile node needs to first wait until it ‘sees’ other mobile node (until it gets inside the transmission range of the other node) for data-relay, and then will be able to relay the data during the period it maintains the connection with the other node. The former metric is called the *inter-meeting time* of the two nodes [8, 16, 35, 17], and the latter is called the *contact time* [17, 21]. These two metrics are critical in determining the delay and capacity of the network, as well as choosing various scheduling/forwarding algorithms. In particular, the inter-meeting time of two nodes is a major component of the end-to-end delay in MANET [15, 13, 14, 31, 32, 33], as it denotes how long it takes to encounter the other mobile node to have any chance to relay/forward the data for communications. Larger inter-meeting time leads to larger end-to-end delay.

There have been several studies on the characteristics of the inter-meeting time in the literature. For example, authors in [14, 31, 13] assume that the inter-meeting time is exponentially distributed so as to make their analysis tractable. This assumption is supported by numerical simulations [32, 34, 13] based on random waypoint mobility model (RWP) [5, 9, 19, 20, 7]. Further, there have been also some theoretical results showing that the first and second moments of the inter-meeting time are bounded above [34, 33] under Brownian motion model on a sphere [12, 32].

However, recent empirical results on the inter-meeting time via real, extensive mobility traces [8, 16, 17, 35] in fact indicate that the tail behavior of the inter-meeting time is far from being exponential, but is close to a power-law [8, 17]. For example, in [17], four distinct sets of data are used to analyze the inter-meeting time. Two of them test WiFi networks (large area) containing thousands of nodes over three or four months (UCSD [28], Dartmouth [16]); another two use a Bluetooth network [17] containing hundreds of nodes in an office/lab environment (small area) over several days. All of these invariably show that the distribution of the inter-meeting time T follows a power-law over a wide range of timescale, i.e., $\mathbb{P}\{T > t\} \sim t^{-\alpha}$ for some constant $\alpha > 0$. More strikingly, there it was also shown that the power-law exponent α is typically less than 1, making even the first moment of the inter-meeting time infinite. This is in sharp contrast to all the results in the current literature where an exponential inter-meeting time distribution is either assumed or numerically verified through various current mobility models.

The above discrepancy between the recent empirical data and the theoretical/simulation results based on current mobility models has motivated several studies in the literature. [8] investigates the effect of power-law inter-meeting time distribution on the system performance (capacity and delay) and calls for new mobility models to produce power-law inter-meeting time; [27] studies the effect of infrastructure and multi-hop transmission on networks with power-law inter-meeting time and calls for new forwarding algorithm to effectively utilize communication opportunities such as the existence of low-delay multi-hop path; [3] proposes a method to generate power-law inter-meeting time using one dimensional random walk model. While these works provide some answers to the effect of power-law inter-meeting time, the following question still remains: *What is the fundamental reason for this discrepancy?* Answer to this question is very important because the disclosure of the fundamental reasons will provide essential guidelines on all related studies in MANETs – modeling of power-law inter-meeting time, analysis of its effect [8, 27], capacity-delay trade-offs [14, 33, 12, 37], and the design and performance analysis of relaying algorithms [14, 32, 8].

In this paper, we first show that the *finite boundary* is one of the key aspects that give rise to the universal appearance of the exponential inter-meeting time distribution in the literature. Indeed, almost all the current mobility models have boundaries or “end-of-the-world”. For example, in the definition of RWP [5, 9, 19, 20, 7], the boundary is necessary because whenever the node changes direction and/or speed, it selects the destination ‘uniformly’ over some bounded simulation area. As for random direction mobility model (RDM) [30] and *i.i.d.* mobility model [37, 26], a boundary is incorporated into their definitions. Similarly, for random walk mobility model (RWM) [10] or Brownian motion on a sphere [32, 12], the boundary is also enforced as all the mobile nodes are constrained to move on a sphere or inside a boundary. Here the *boundary* may be of different types including ‘reflective’ or ‘wrapping’ as defined in [7], which converts a 2-D rectangular simulation area into a torus-shaped area.

In particular, we rigorously prove that the tail of the inter-meeting time between two independent mobile nodes¹ decays *at least exponentially fast* for any RWP model and for any RWM (or RDM) as long as the boundary is finite, i.e., the mobility model is defined on a bounded domain. Our assumptions are quite general in that they are satisfied by nearly all the current mobility models in the literature. Our results readily show the finiteness of all the moments of the inter-meeting time (much stronger than the finiteness of only the first and second moments as shown in [34]), and provide analytical support on the simulation-based studies of the inter-meeting time under the current mobility models with boundaries [32, 34, 13].

Our theoretical results imply that the finite boundary, among others, is the key determining factor of the exponential decay in the inter-meeting time distribution, and thus potentially undermines our understanding of the performance of MANET. Moreover, we prove that removing the

¹Mobile nodes may be dependent if they belong to the same ‘group’ and follow similar paths, or belong to different groups whose pathways rarely overlap. The inter-meeting time in such cases may show very different characteristics, which is beyond the scope of this paper.

boundary from 2-D RWM readily gives the power-law distribution of the inter-meeting time.² We also discuss the relationship between the size of the boundary and the relevant timescale of interest and their effect on the inter-meeting time distribution, and provide simulation results to support this relationship.

The rest of the paper is organized as follows. Section 2 first presents related work on empirical observation of the power-law inter-meeting time distribution, and then provides several definitions and technical results to be used later on. In Section 3, we prove that mobility models with finite boundary always yield exponential inter-meeting time distribution at least in an asymptotic sense. In Section 4, we show that mobility models in an unbounded space necessarily produce power-law decay of the inter-meeting time. In Section 5, we provide some discussion on the relationship between the size of the boundary and timescale of interest, and their effect on the inter-meeting time. Some simulation results are presented in Section 6 to support our theoretical findings and discussion on time/space scaling. We finally conclude in Section 7.

2. PRELIMINARIES

2.1 Empirical Observation of Power-law

The power-law behavior of the inter-meeting time distribution has been reported through various data sets under different scenarios. In this section, we provide brief summary on how those data sets have been collected and interpreted in the context of the inter-meeting time distribution, while referring to [17, 8, 27] for more details.

Most data sets available for inter-meeting time study in the literature can be classified into access point (AP) based or Bluetooth-based ones. AP-based data sets [28, 16] keep track of the time instants at which a mobile node gets associated with an AP and typically record activities (AP logs) of a large number of mobile nodes for a long period of time. Two nodes are assumed to ‘meet’ only if both of them are in the range of the same AP. In view of the inter-meeting time of mobile nodes, some inaccuracies are inevitable in these AP-based data sets since (i) two nodes within the range of each other may not be within the range of the same AP, i.e., the duration of the inter-meeting time could be overestimated, and (ii) two nodes within the range of the same AP may be out of their communication ranges as well, i.e., the inter-meeting time could also be underestimated.

On the other hand, Bluetooth-based data sets record time instants at which a node is in contact with other nodes, along with the ID of the node in contact [17, 8, 27]. While the number of nodes and the duration of the activities are typically smaller than those of AP-based ones,³ Bluetooth-based data provide more accurate information on the inter-meeting time, as they are free of any reference to APs and directly keep track of contact/inter-contact events. Nevertheless, in all AP-based and Bluetooth-based data sets, the power-law

²Similar suggestion has been made in [3] via 1-D random walk, while our example in Section 4 is in 2-D.

³For example, the UCSD data set [28] includes more than 195 thousands contacts collected from 273 participating devices over 16 days, while the Infocom 2005 Bluetooth-based data set includes about 7 thousands contacts collected from 41 dedicated iMote devices over 3 days.

behavior of the inter-meeting time has been shown to persist over from several hours up to even days.

2.2 Assumption and Definitions

In this section we collect basic assumptions, notations and several definitions of the metric of interest that will be used throughout the paper. We also provide some basic properties associated with those definitions and notations.

We consider two mobile nodes A and B, each of which moves according to some mobility model in $\Omega \subset \mathbb{R}^2$. The domain Ω (the entire ‘world’ for the mobile nodes) can be bounded or unbounded depending on our choice of mobility models in sequel. Let $A(t), B(t) \in \Omega$ be the position of the node A and B at time t , respectively. We will assume that $A(t)$ and $B(t)$ are independent unless otherwise specified. As our main interest lies in the statistics of the inter-meeting time of two nodes, we find the following definition useful.

DEFINITION 1. Let $\{S(t) \in \Omega\}_{t \geq 0}$ be a collection of subsets of Ω . The first passage time (FPT) T_F of $A(t)$ to $\{S(t)\}$ is defined as

$$T_F \triangleq \inf_{t > 0} \{t : A(t) \in S(t)\}.$$

given that $A(0) \notin S(0)$. \square

ASSUMPTION 1. Two mobile nodes can communicate with each other whenever they are within a distance of d , the transmission range of the mobile node. \square

REMARK 1. Assumption 1 is reasonable if the density of the mobile nodes is not high and the interference from other mobile nodes are negligible. This is especially the case in sparse mobile networks or delay/disruption tolerant networks (DTNs) [11, 21, 27, 6, 23, 17]. In case of high level of density of mobile nodes, our assumption might be rather optimistic and would lead to underestimation of the actual inter-meeting time, since two nodes may not be able to communicate due to interference even when they are nearby. Still, even for such a highly populated MANET, our subsequent analysis can also be applied to different mobile ‘groups’ whose density is usually much lower than that of mobile nodes.

DEFINITION 2. The inter-meeting time T_I of nodes A and B is defined as

$$T_I \triangleq \inf_{t > 0} \{t : \|A(t) - B(t)\| \leq d\}, \quad (1)$$

given that $\|A(0) - B(0)\| = d$ and $\|A(0^+) - B(0^+)\| > d$. Here, $\|\cdot\|$ is the Euclidian norm in 2-D. \square

Define by $\mathcal{N}_B(t)$ a set of points that can be reached by node B at time t . Specifically,

$$\mathcal{N}_B(t) \triangleq \{s \in \Omega : \|s - B(t)\| \leq d\}. \quad (2)$$

Then, the inter-meeting time becomes the first passage time of $A(t)$ to $\{\mathcal{N}_B(t)\}$, given that $A(0) \in \mathcal{N}_B(0)$ and $A(0^+) \notin \mathcal{N}_B(0^+)$.

As we will be interested in the behavior of the tail of the inter-meeting time distribution, it is advantageous to introduce the following definition.

DEFINITION 3. Let \mathcal{P} be a collection of positive random variables whose complementary distribution decay at least

exponentially fast. Specifically, we write $X \in \mathcal{P}$ if there exists a constant $c > 0$ such that

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log \mathbb{P}\{X > t\} = -c < 0. \quad (3)$$

Note that (3) implies $\mathbb{P}\{X > t\} \leq \exp(-ct + o(t))$ where $o(t)/t \rightarrow 0$ as $t \rightarrow \infty$. Or, equivalently, $\mathbb{P}\{X > t\} = \exp(-c[t + o(t)]/t) \leq Ke^{-ct}$ for some $K > 0$ and for all sufficiently large t , indicating that the tail of X decays at least exponentially fast. With this definition, we immediately obtain the following properties that will be useful later.

(P1) If $X, Y \in \mathcal{P}$, then $aX + bY \in \mathcal{P}$ for any positive constants a, b .

(P2) If $X \in \mathcal{P}$, then $X_e \in \mathcal{P}$, where X_e has the equilibrium distribution of X , i.e.,

$$F_e(x) = \mathbb{P}\{X_e \leq x\} = \int_0^x (1 - F(u)) du / \mathbb{E}\{X\}, \quad (4)$$

where $F(x) = \mathbb{P}\{X \leq x\}$.

Property P1 can be seen by noting that for any (possibly dependent) random variables X and Y ,

$$\mathbb{P}\{aX + bY > t\} \leq \mathbb{P}\{X > t/(2a)\} + \mathbb{P}\{Y > t/(2b)\}. \quad (5)$$

Thus, if $X \in \mathcal{P}$ with constant $c_1 > 0$ and $Y \in \mathcal{P}$ with $c_2 > 0$, it follows that

$$\limsup_{t \rightarrow \infty} \frac{1}{t} \log \mathbb{P}\{X + Y > t\} = -\min\left\{\frac{c_1}{2a}, \frac{c_2}{2b}\right\} < 0.$$

P2 follows since

$$\begin{aligned} \mathbb{P}\{X_e > t\} &= \frac{1}{\mathbb{E}\{X\}} \int_t^\infty \mathbb{P}\{X > x\} dx \\ &\leq \frac{1}{\mathbb{E}\{X\}} \int_t^\infty Ke^{-cx} dx = \frac{K}{c\mathbb{E}\{X\}} e^{-ct} \end{aligned}$$

for all sufficiently large t . Further, $X \in \mathcal{P}$ ensures that all the moments of X are finite, thus its moment generating function is well-defined, i.e., $\mathbb{E}\{e^{\theta X}\} < \infty$ for some $\theta > 0$.

3. INTER-MEETING TIME WITH EXPONENTIAL TAIL

In this section we are interested in mobility models with finite boundaries, i.e., $\Omega \in \mathbb{R}^2$ is bounded, and rigorously prove that mobility models such as random waypoint mobility models (RWP) and random walk models (RWM), defined on a bounded domain Ω necessarily result in an asymptotic exponential tail of the inter-meeting time. We will consider the case of unbounded Ω (e.g., $\Omega = \mathbb{R}^2$) later in Section 4.

3.1 RWP Models

In the RWP model [18], a node first selects a random waypoint as its destination uniformly in a convex bounded region Ω , and a speed V uniformly from $[v_{min}, v_{max}]$ ($0 < v_{min} \leq v_{max}$), and then moves to its destination at its chosen speed. After it reaches the destination, it pauses for a random amount of time and then selects a new destination and speed, and repeat the whole procedure independently. Denote the time instant at which a node pauses and chooses a new destination as *joint time* and the path of a node consisting of a straight line segment as one *leg* [18]. The speed of node A is chosen uniformly from $[v_{min}^A, v_{max}^A]$. Let τ_n^A be the duration of the n^{th} pause time of node A. Similarly we define τ_n^B for node B.

We first consider the traditional RWP model where all the pause times are zero and mathematically prove that the inter-meeting time decays at least exponentially fast. Later in this section, we generalize the result to include the case of non-zero random (possibly infinite) pause time. Throughout the section we assume that nodes A and B are independent, but we do allow that the speed and pause time distributions are different for different nodes.

RWP with Zero Pause Time: First, consider the zero pause time case. Let $\tau_n^A = \tau_n^B = 0$ for all $n = 1, 2, \dots$. Then we have the following result.

PROPOSITION 1. *There exists constant $c > 0$ such that*

$$\mathbb{P}\{T_I > t\} \leq e^{-ct},$$

for all sufficiently large t . \square

PROOF. Let $D < \infty$ be the ‘diameter’ of Ω or the maximum length of a line segment (leg) in Ω . Let

$$\zeta = 2D / \min\{v_{min}^A, v_{min}^B\} > 0,$$

which is well defined since $v_{min} > 0$ for both nodes and $D < \infty$. Note that ζ is the longest time it takes for both nodes to finish two legs. In other words, during the time interval of ζ , both nodes must have jumped at least twice. Hence, ζ can be interpreted as a *renewal* interval for nodes A and B since for any $t > 0$, $\{A(s); s \leq t\}$ and $\{A(u+\zeta); u \geq t\}$ are independent (similarly for node B), i.e., after ζ seconds, nodes A and B completely forget where they were ζ seconds ago [18].

Let π_A and π_B be the stationary node distributions of A and B, respectively, and let $X(t) = (A(t), B(t)) \in \Omega^2$. Without loss of generality, we assume that the initial distribution of $X(t)$ is chosen as its stationary distribution $\pi_A \times \pi_B$. One immediate consequence is that the joint node distribution of A and B becomes stationary, i.e., $\mathbb{P}\{A(t) \in S_1, B(t) \in S_2\} = \pi_A(S_1)\pi_B(S_2)$ for all $t > 0$ and $S_1, S_2 \subset \Omega$.

Let $\mathcal{E}_t = \{\|A(t) - B(t)\| > d\}$ be the event that two nodes are ‘out-of-range’ at time t . From the stationarity assumption, $\mathbb{P}\{\mathcal{E}_t\}$ does not depend on t . Since the stationary node distribution of the RWP is non-zero on any point of Ω [18], for any given $B(t) = B \in \Omega$, we see that there exists a positive constant f (may depend on d and the shape of Ω) such that $\pi_A(\mathcal{N}_B(t)) \geq f > 0$ where $\mathcal{N}_B(t)$ is from (2). Thus, by taking average again with respect to π_B , we have

$$\mathbb{P}\{\mathcal{E}_t\} \leq 1 - f \triangleq \rho < 1. \quad (6)$$

Observe that for the inter-meeting time T_I , we have

$$\{T_I > t\} \equiv \bigcap_{s=0+}^t \mathcal{E}_s \subset \bigcap_{s \in \mathcal{I}_t} \mathcal{E}_s \quad (7)$$

for any subset (index set) $\mathcal{I}_t \subset (0, t]$. Now, we choose the index set \mathcal{I}_t as $\mathcal{I}_t = \{\zeta, 2\zeta, \dots, \lfloor \frac{t}{\zeta} \rfloor \zeta\}$. Since ζ is the ‘renewal’ interval for both nodes, it follows that $X(t_i)$ ($i = 1, 2, \dots$) are all independent whenever $t_i \in \mathcal{I}_t$ and $t_i \neq t_j$. Thus, \mathcal{E}_{t_i} for different t_i in the index set \mathcal{I}_t are also independent. Hence, from (7),

$$\mathbb{P}\{T_I > t\} \leq \mathbb{P}\left\{\bigcap_{s \in \mathcal{I}_t} \mathcal{E}_s\right\} = \prod_{s \in \mathcal{I}_t} \mathbb{P}\{\mathcal{E}_s\} \leq \rho^{\lfloor \frac{t}{\zeta} \rfloor} \sim e^{-ct}, \quad (8)$$

where $c = -(\log \rho) / \zeta > 0$ from $0 < \rho < 1$ and $0 < \zeta < \infty$. This completes the proof. \square

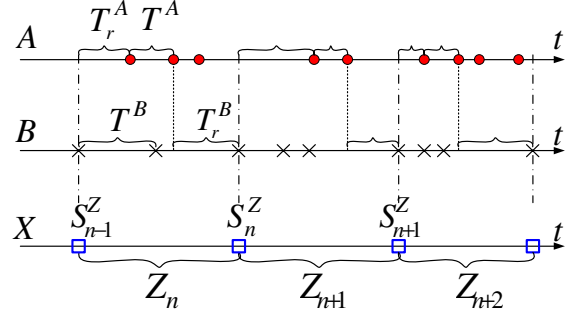


Figure 1: Illustration for the construction of Z_n . Start from S_{n-1}^Z , first skip two consecutive ‘renewal’ points of node A and then wait for the next following renewal point of node B. This defines Z_n . T_r^A and T_r^B are ‘residual life’ of node A and B, respectively.

REMARK 2. *Proposition 1 can be immediately extended to the case of non-zero pause time as long as it is bounded above. If $\max\{\tau_n^A, \tau_n^B\} \leq M < \infty$, we just need to redefine the renewal interval as $\zeta' = \zeta + 2M$ in the above proof.*

RWP with Random Pause Time: We now consider RWP models with non-zero random pause time. Let V^A, V^B and τ^A, τ^B be the speed and pause time of two independent nodes A and B, respectively. Similarly as before, we define a sequence of ‘renewal’ intervals for node A as

$$T_n^A \triangleq \frac{D}{V_{2n-1}^A} + \tau_{2n-1}^A + \frac{D}{V_{2n}^A} + \tau_{2n}^A, \quad \text{for } n = 1, 2, \dots,$$

where $D < \infty$ is the diameter of Ω . In words, T_n^A is the longest time it takes for node A to finish two legs including two pause times. Then, as before, for each T_n^A , $\{A(s); s \leq t\}$ and $\{A(u+T_n^A); u > t\}$ are independent. Since $\{V_n^A, n \geq 1\}$ are *i.i.d.* (so are $\{\tau_n^A, n \geq 1\}$), it follows that $\{T_n^A, n \geq 1\}$ are also *i.i.d.* Similarly define T_n^B for node B.

Define $X(t) \triangleq (A(t), B(t)) \in \Omega^2$ and assume that the system is in the steady-state with its stationary distribution. Then, along the same line in the proof of Proposition 1, we have for some $0 < \hat{\rho} < 1$,

$$\mathbb{P}\{T_I > t\} \leq \hat{\rho}^{|T_t|}, \quad (9)$$

where $|S|$ is the cardinality of a set S and $T_t = \{t_1, t_2, \dots, t_{|T_t|}\} \subset (0, t]$ is an index set with the following property:

$$\{X(t_i), i = 1, 2, \dots, |T_t|\} \text{ are independent.} \quad (10)$$

The difficulty here lies in the fact that, in contrast to Proposition 1, there is no finite ζ such that $X(t)$ and $X(t+\zeta)$ become independent. In addition, even if T_n^A forms a ‘renewal’ sequence for $A(t)$ (T_n^B for $B(t)$), this does not mean that $X(t)$ is also ‘renewing’ on any such sequence. For instance, after a ‘renewal’ of node A, it could be that the pause time of node B is exceptionally long so that the node A still has to renew itself possibly an infinite number of times before it ‘sees’ a renewal of node B.

In what follows, we will construct a random index set \mathcal{I}_t satisfying (10). For notational simplicities, we use T^A and T^B to denote the distribution of T_n^A and T_n^B , T_e^A and T_e^B to denote the equilibrium distribution of T^A and T^B (see (4)), respectively, whenever no ambiguity arises.

Set $t = 0$ as one of the ‘renewal’ point of node B. We will inductively construct a sequence of Z_n , $n = 1, 2, \dots$, where

each Z_n is a sum of non-overlapping T_k^B . Define

$$S_n^Z = \sum_{k=1}^n Z_k$$

and $S_0^Z = 0$. Starting from S_{n-1}^Z ($n \geq 1$), we first skip two consecutive ‘renewal’ points of node A and then mark the next following renewal point of node B. Let t' be this marked point. Then, we set $Z_n \triangleq t' - S_{n-1}^Z$ and $t' = S_n^Z$. See Figure 1 for illustration. Notice that Z_n consists of three parts: (i) the residual life of node A (T_r^A) evaluated at S_{n-1}^Z (time from S_{n-1}^Z until the first renewal point of node A), (ii) a renewal interval of length T_k^A for some k , and (iii) the residual life of node B (T_r^B) evaluated at the second renewal point of node A (from the second renewal point of node A until S_n^Z). From this construction, we have the following:

Claim 1: $\{Z_{2k-1}\}_{k \geq 1}$ are independent. So are $\{Z_{2k}\}_{k \geq 1}$.

Claim 1 immediately follows since for each n , the first component of Z_{n+2} (residual life of node A) and the last component of Z_n (residual life of node B) are disjoint and separated apart by at least one renewal interval of node A and B (see Figure 1).

In order to proceed, we need the following technical assumption.

ASSUMPTION 2. Let $\mu_k = \mathbb{E}\{Z_k\} < \infty$. Then,

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \mu_{2k-1} = \tilde{\mu}_1 < \infty. \quad (11)$$

Further, there exists $\theta_0 > 0$ such that for each $\theta \in [0, \theta_0)$, we have

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n \log \mathbb{E} \left\{ e^{\theta(Z_{2k-1} - \mu_{2k-1})} \right\} = \Lambda_1(\theta) < \infty, \quad (12)$$

where $\Lambda_1(\theta)$ is well-defined and differentiable at $\theta = 0$. Similarly for $\tilde{\mu}_2$ and $\Lambda_2(\theta)$ with indices in (11) and (12) replaced by even numbers. \square

REMARK 3. Assumption 2 is quite reasonable since (11) and (12) require that $\{Z_{2k-1}\}_{k \geq 1}$ have similar distribution and the moment generating function of each zero-mean random variables $Z_k - \mu_k$ be finite, only in the asymptotic sense. For example, if each of the three components belongs to \mathcal{P} ,⁴ then from property (P1), it readily follows that $Z_n \in \mathcal{P}$, which guarantees the existence of the moment generating function. One special case can be where T^A and T^B are both exponentially distributed, for which T_r^A and T_r^B are also exponentially distributed (due to memoryless property) and thus Assumption 2 is automatically satisfied. \square

We now state our main result.

THEOREM 1. Under Assumption 2, we have $T_I \in \mathcal{P}$, i.e., the inter-meeting time of nodes A and B decays at least exponentially fast. \square

PROOF. Similarly as in the proof of Proposition 1, we take one ‘snapshot’ of $X(t)$ per each renewal interval Z_{2n}

⁴If $T^A, T^B \in \mathcal{P}$ and the system is in the steady-state that $T^A \stackrel{d}{=} T_e^A$ (same for T^B), then properties (P1) and (P2) ensure that $Z \stackrel{d}{=} T_e^A + T^A + T_e^B \in \mathcal{P}$.

($n = 1, 2, \dots$). To be specific, we construct a random index set $\mathcal{I}_t = \{t_1, t_2, \dots\}$ as follows: choose $t_1 \in [S_1^Z, S_2^Z]$, $t_2 \in [S_3^Z, S_4^Z]$, and in general, $t_j \in [S_{2j-1}^Z, S_{2j}^Z]$. Repeat this procedure until S_{2n}^Z becomes larger than t . Then, for any $i < j$, $[t_i, t_j]$ contains at least one full renewal interval of node A and B, thus $X(t_i)$ and $X(t_j)$ are independent. In this way, it is straightforward to see that $\{X(t_i), i = 1, 2, \dots\}$ are independent (and actually identically distributed from the stationarity of $X(t)$).

Let $N(t) \triangleq |\mathcal{I}_t|$ be a counting process that tells us the number of independent ‘snapshots’ of $X(u) = (A(u), B(u))$ during $[0, t]$. By conditioning on $N(t)$ and from (9), we have

$$\mathbb{P}\{T_I > t\} \leq \mathbb{E}\{\hat{\rho}^{N(t)}\} = \mathbb{E}\{e^{-\xi N(t)}\}, \quad (13)$$

where $\xi = -\log \hat{\rho} > 0$. Note that, for any constant $\beta > 0$,

$$\begin{aligned} \mathbb{E}\{e^{-\xi N(t)}\} &= \mathbb{E}\{e^{-\xi N(t)} 1_{\{N(t) \leq \beta t\}}\} + \mathbb{E}\{e^{-\xi N(t)} 1_{\{N(t) > \beta t\}}\} \\ &\leq \mathbb{P}\{N(t) \leq \beta t\} + e^{-\xi \beta t}. \end{aligned} \quad (14)$$

A closer look at the way we construct \mathcal{I}_t reveals that

$$\{N(t) \leq \beta t\} \equiv \{S_{2\beta t}^Z \geq t\}$$

assuming that βt takes on integer value without loss of generality. Thus, for all sufficiently large t , we have

$$\begin{aligned} \mathbb{P}\{N(t) \leq \beta t\} &= \mathbb{P}\{S_{2\beta t}^Z \geq t\} = \mathbb{P}\{S_{2\beta t}^Z \geq \frac{n}{\beta}\} \quad (\text{set } n = \beta t) \\ &\leq \mathbb{P}\left\{\sum_{k=1}^n Z_{2k-1} \geq \frac{n}{2\beta}\right\} + \mathbb{P}\left\{\sum_{k=1}^n Z_{2k} \geq \frac{n}{2\beta}\right\} \end{aligned} \quad (15)$$

where (15) is from (5) with $a = b = 1$. For the first term in (15), note first that from the assumption in (11), for any given $\epsilon > 0$, we have

$$\left| \frac{1}{n} \sum_{k=1}^n \mu_{2k-1} - \tilde{\mu}_1 \right| < \epsilon \quad (16)$$

for all sufficiently large n . We then choose $\beta > 0$ such that

$$\alpha \triangleq \frac{1}{2\beta} - (\tilde{\mu}_1 + \epsilon) > 0. \quad (17)$$

Then, from (16) and (17), it follows that

$$\frac{1}{2\beta} = \alpha + \tilde{\mu}_1 + \epsilon > \alpha + \frac{1}{n} \sum_{k=1}^n \mu_{2k-1} \quad (18)$$

for all sufficiently large n .

Rewriting the first term in (15) gives, for any $\theta > 0$ and for all sufficiently large n ,

$$\mathbb{P}\left\{\sum_{k=1}^n Z_{2k-1} \geq \frac{n}{2\beta}\right\} \leq \mathbb{P}\left\{\sum_{k=1}^n (Z_{2k-1} - \mu_{2k-1}) \geq \alpha n\right\} \quad (19)$$

$$\leq e^{-\theta \alpha n} \mathbb{E}\left\{\exp\left(\theta \sum_{k=1}^n [Z_{2k-1} - \mu_{2k-1}]\right)\right\} \quad (20)$$

$$= \exp\left(-n \left[\alpha \theta - \frac{1}{n} \sum_{k=1}^n \log \mathbb{E}\{e^{\theta(Z_{2k-1} - \mu_{2k-1})}\}\right]\right), \quad (21)$$

where (19) is from (18), (20) is from Markov’s inequality, and (21) is from independence of $\{Z_{2k-1}\}_{k \geq 1}$ (Claim 1). We then take log in both sides of (21), divide by n , and take limit with respect to n . By optimizing the resulting

expression over $0 < \theta < \theta_0$, from the assumption in (12), we have

$$\lim_{n \rightarrow \infty} \frac{1}{n} \log \mathbb{P} \left\{ \sum_{k=1}^n Z_{2k-1} \geq \frac{n}{2\beta} \right\} \leq -I(\alpha) \quad (22)$$

where

$$I(x) = \sup_{0 < \theta < \theta_0} [x\theta - \Lambda_1(\theta)].$$

Note that the function $I(x)$ is a well-defined convex function from Assumption 2. Further, since $\Lambda_1(0) = 0$ and $\lambda'_1(0) = 0$ (from $\mathbb{E}\{Z_k - \mu_k\} = 0$), we have $I(\alpha) > 0$ from our choice of $\alpha > 0$ in (17). Similarly, we can repeat the same lines for the second term in (15). This completes the proof of Theorem 1. \square

REMARK 4. *The exponential tail behavior of the inter-meeting time of two nodes moving according to RWP model has been mentioned in [34]. However, while [34] provides only simulation results, we rigorously prove the results. Our theoretical result is quite general in that it applies to not only the bounded pause time case, but also the random, unbounded pause time case (see Assumption 2). Moreover, two users under discussion are not required to have the same stationary node distribution.*

3.2 Random Walk Models (RWM)

In the current literature studying the RWM [12, 33], commonly a discrete-time, discrete-space 2-D model is used: A square (the bounded area) is divided into multiple sub-squares, called cells, and time is divided into slots of equal duration. At each time slot a node is in and can be only in one cell. The initial position of a node is uniformly chosen from all cells. At the beginning of each time slot, the node jumps from its current cell to one of its adjacent cells with equal probability. Moreover, the boundary can be wrapping or reflecting (see Figure 2).

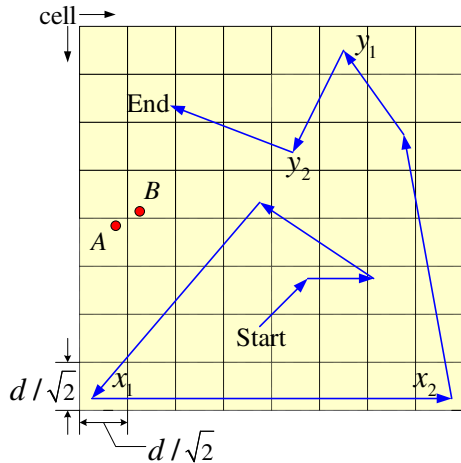


Figure 2: A trajectory of our RWM model: The node can jump from a cell to any other cell with certain probability, and the boundary condition can be arbitrary ($x_1 \rightarrow x_2$: wrapping; $y_1 \rightarrow y_2$: reflecting).

In this section, we consider a more general RWM. The bounded area is assumed to be a unit square. Divide the

square into $N = 1/[\frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}}]$ sub-squares (cell)⁵, each of which with area $\frac{d}{\sqrt{2}} \times \frac{d}{\sqrt{2}}$ as shown in Figure 2. The initial position of a node is chosen uniformly from N cells. We number the cells by $1, 2, \dots, N$: the cell on the n^{th} row and m^{th} column is assigned the number $i = (n-1)\sqrt{N} + m$, where $n, m = 1, 2, \dots, \sqrt{N}$ and $i = 1, 2, \dots, N$. At the beginning of each time slot, the node jumps from the i^{th} cell to the j^{th} cell with probability $p(i, j) \in [0, 1]$. This model is very general in the sense that: (i) in any position, the node can jump to any other cell (including the current cell) with any preassigned probability, which obviously includes the model in [12, 33] where the node can only jump to one of its adjacent cells with equal probability, and (ii) we allow any arbitrary boundary condition including wrapping, reflecting, etc., by assigning different jumping probabilities for the cells on the boundary. (See Figure 2 for example.)

This general RWM can be studied using a Markov Chain (MC) with N states and its transition matrix $P = \{p_{ij}\}$ where $p_{ij} = p(i, j)$, i.e., the probability of jumping from state i to j . In the following, we will give our main result on the inter-meeting time of a pair of nodes, in which only one of the two nodes is required to follow the RWM described above and the mobility model of the other node can be completely arbitrary.

To avoid trivialities, the MC is assumed to be irreducible, i.e., any single node in this finite system can go to anywhere in a finite number of steps. Still, this alone is not sufficient to establish a well-defined mobility model. For example, define a mobility model such that the node can only jump from cell i to cell $i+1$ (when $i=N$, the user can only jump to cell 1). Obviously this MC is irreducible, but it is not a good mobility model. The introduction of a mobility model is to emulate the mobility pattern in real life in an abstract and simple way. In reality, if an obstacle is present, the moving object (people, bus, etc.) will simply bypass it and continue the movement in the system, rather than get stuck on it.

From the viewpoint of graph theory, the mobility model can be looked as a directed graph. Each cell corresponds to a vertex, and there is a directed edge from vertex i to vertex j if and only if $p_{ij} > 0$. Hence, the irreducibility actually ensures that the graph is connected. However, how strong is the connectivity? One fundamental way to test it is to delete one vertex; if the graph is still connected after the deletion of any one vertex, then we say that the graph is 2-connected. For example, the mobility model example given above is connected, but not 2-connected. In view of this, we impose the following assumption to avoid trivial models.

ASSUMPTION 3. *After we delete any single state from the MC model, the resulting state space is still a communicating class.* \square

Note that we are interested in the tail behavior of the inter-meeting time, so we surely want to rule out any possibility that two nodes never meet. For example, if nodes A and B somehow *conspire together* not to meet each other forever (e.g., node B always move ‘parallel’ to node A), the inter-meeting time becomes infinite with probability 1. In order to rule out this possibility, we also need the following technical assumption.

⁵We assume \sqrt{N} is an integer without loss of generality.

ASSUMPTION 4. For any possible trajectory of node B, node A eventually meets node B with positive probability. Specifically, there exists $M < \infty$ such that $\mathbb{P}\{T_I > M\} < 1$ regardless of the trajectory of node B, i.e., $\mathbb{P}\{T_I > M\} < 1$ is true for any trajectories. \square

REMARK 5. Assumptions 3 and 4 ensure that (i) there exist at least two different paths between any two cells and (ii) it is impossible for node B to get away from node A forever even if it chooses its ‘optimal’ path to stay away from node A (no conspiracy). Note that all existing versions of RWM [7, 12, 33] readily satisfy Assumptions 3 and 4. \square

To proceed, we present the following definitions and lemma:

DEFINITION 4. Let $\|\cdot\|_\infty$ be the norm of a matrix $K = \{k_{ij}\}$ defined by $\|K\|_\infty \triangleq \max_i \sum_j |k_{ij}|$, i.e., the largest absolute row sum. For any k -dimensional vector $\vec{x} = \{x_1, \dots, x_k\}$ and $n \in \mathbb{N}$, we define n -norm of \vec{x} as $\|\vec{x}\|_n = (\sum_{i=1}^k |x_i|^n)^{1/n}$. \square

DEFINITION 5. A finite set of matrices $C_m = \{c_{ij}^m\} \in \mathbb{R}^{N \times N}$ where $m \in \{1, \dots, L\}$ is said to be a sub-stochastic matrix set if there exists a finite integer M such that for any integer set $\{h_1, \dots, h_M\}$ with $h_i \in \{1, \dots, L\}$ and $i \in \{1, \dots, M\}$, $\|C_{h_1} \dots C_{h_M}\|_\infty < 1$. \square

LEMMA 1. Let $\{P_m\} \in \mathbb{R}^{N \times N}$ ($m = 1, 2, \dots, M$) be a sub-stochastic matrix set. Then, for any index set $\mathcal{I} = \{n_1, n_2, \dots\}$ where $n_i \in \{1, \dots, M\}$ and $i \in \mathbb{N}$, there exist constants $c > 0$ and $0 < q < 1$ (independently of the choice of \mathcal{I}) such that for any $\vec{x} \in \mathbb{R}^{1 \times N}$, the product of any k ($k \in \mathbb{N}$) matrices $A(\mathcal{I}, k) = P_{n_1} P_{n_2} \dots P_{n_k}$ satisfies

$$\|\vec{x}A(\mathcal{I}, k)\|_n \leq cq^k \|\vec{x}\|_n \quad \text{for all sufficiently large } k.$$

PROOF. See Appendix. \square

We now state our main result.

THEOREM 2. Suppose that node A moves according to the RWM described above satisfying Assumptions 3 and 4. Then, there exists constant $\gamma > 0$ such that $\mathbb{P}\{T_I > t\} \leq e^{-\gamma t}$ for all sufficiently large t .

REMARK 6. We require that only one of the nodes (node A) moves according to the RWM. This means that the other node (node B) can move according to any other arbitrary mobility models defined on a bounded domain such as RWP, random direction models, etc., as long as two nodes eventually meet with non-zero probability.

PROOF. Since node B moves according to some arbitrary mobility model, the only information we know about node B’s movement is its path-wise position at $t = 0, 1, 2, \dots$. Let \mathcal{T} be the set of all possible trajectories of node B satisfying Assumptions 4. Without loss of generality, assume A and B are in the communication range just before time 0 and $A(0) = 1 \neq B(0)$. (If $A(0) \neq 1$, simply renumber the cells.) Let $T_F^{\mathcal{B}}$ be the FPT of A to the trajectory $\mathcal{B} \in \mathcal{T}$ as in Definition 1. Then, we have

$$\mathbb{P}\{T_I > t\} = \sum_{\mathcal{B} \in \mathcal{T}} \mathbb{P}\{T_F^{\mathcal{B}} > t\} \mathbb{P}\{\mathcal{B}\} \quad \text{for all } t \geq 1. \quad (23)$$

In the following, we fix $\mathcal{B} \triangleq \{B(t) = k_t, t \in \mathbb{N}\}$ ($k_t \in \{1, 2, \dots, N\}$) to show that $\mathbb{P}\{T_F^{\mathcal{B}} > t\}$ is uniformly upper-bounded by $c \exp(-\gamma t)$ where $c > 0$ and $\gamma > 0$, i.e., for any

$\mathcal{B} \in \mathcal{T}$, $\mathbb{P}\{T_F^{\mathcal{B}} > t\} \leq c \exp(-\gamma t)$. For the simplicity of expression, we use T_F in stead of $T_F^{\mathcal{B}}$ because now we are discussing fixed trajectory \mathcal{B} .

Let $P = \{p_{ij}\}$ be the transition matrix of the Markov chain for $A(t)$ (position of node A at time t) where p_{ij} is the probability of jumping from cell i to cell j in one step. We then define a set of matrices $\{P_i, i = 1, 2, \dots, N\}$ by setting the i^{th} column of P to 0 and keeping all the other elements the same.

From $A(0) = 1$, the initial state vector becomes $\vec{x}_0 = [1 \ 0 \ \dots \ 0]$. Then $\mathbb{P}\{T_F > 1\}$ is the probability that node A jumps from state 1 to any state other than $B(1) = k_1$, i.e.,

$$\mathbb{P}\{T_F > 1\} = \sum_{1 \leq j \leq N, j \neq k_1} p_{1j}. \quad (24)$$

Define a matrix $Q^{(1)} \in \mathbb{R}^{N \times N}$ as $Q^{(1)} = P_{k_1}$ and denote by $q_{ij}^{(1)}$ the i^{th} row and j^{th} column element of $Q^{(1)}$. Then, from (24), it follows that $\mathbb{P}\{T_F > 1\} = \sum_{j=1}^N q_{1j}^{(1)} = \vec{x}_0 Q^{(1)} \mathbf{1}$, where $\mathbf{1} = [1 \ 1 \ \dots \ 1]^T$ is the N -dimensional column vector whose elements are all 1. Following the similar line, we define $Q^{(t)} = Q^{(t-1)} P_{k_t}$ for all $t \geq 2$. Then we can show by induction that

$$\mathbb{P}\{T_F > t\} = \sum_{j=1}^N q_{1j}^{(t)} = \vec{x}_0 Q^{(t)} \mathbf{1}, \quad (25)$$

where $q_{1j}^{(t)}$ is the first row and j^{th} column element of the following matrix $Q^{(t)}$:

$$Q^{(t)} = P_{k_1} P_{k_2} \dots P_{k_t}, \quad t \geq 1. \quad (26)$$

Consequently, if we can prove that for any given trajectory \mathcal{B} of node B, there exists constant $c < \infty$ and $\gamma > 0$ (independently of \mathcal{B}) such that $\vec{x}_0 Q^{(t)} \mathbf{1} \leq c e^{-\gamma t}$ for all sufficiently large t , then from (23) and (25), we are done.

In what follows, we will show that $\{P_i, i = 1, \dots, N\}$ is a sub-stochastic matrix set as defined in Definition 5. From (25), $\vec{x}_0 Q^{(t)} \mathbf{1}$ is the sum of the first row of $Q^{(t)}$, or equivalently, the probability that node A starts from cell 1 ($A(0) = 1$) and never meets node B (with the trajectory \mathcal{B}) up to time t . Similarly, it can be shown that the sum of the i^{th} row of $Q^{(t)}$ is the probability that node A starts from cell i ($A(0) = i$) and never meets B with the trajectory \mathcal{B} up to time t . Thus, from Assumption 4, there exists $0 < M < \infty$ such that for any $t \geq M$, $\sum_j q_{ij}^{(t)} = \mathbb{P}\{T_F > t\} \leq \mathbb{P}\{T_F > M\} < 1$, for any trajectory \mathcal{B} and any $i \in \{1, 2, \dots, N\}$. Subsequently, for all $t \geq M$,

$$\|P_{k_1} P_{k_2} \dots P_{k_t}\|_\infty = \|Q^{(t)}\|_\infty = \max_i \sum_j |q_{ij}^{(t)}| \quad (27)$$

$$= \max_i \sum_j q_{ij}^{(t)} < 1, \quad (28)$$

where (27) is from (26) and $\|\cdot\|_\infty$ norm is from Definition 4, and the equality in (28) is from $q_{ij}^{(t)} \geq 0$.

Note that (28) is true for any trajectory \mathcal{B} of node B. In particular, choose $\mathcal{B} = \{B(t) = k_t\}$ where $k_t \in \{1, 2, \dots, N\}$ and $t \in \{1, \dots, M\}$ and M is the finite integer as in Assumption 4. Hence, from Definition 5, $\{P_i, i = 1, \dots, N\}$ is a sub-stochastic matrix set.

In consequence, from Lemma 1, there exists constant $c > 0$ and $0 < q < 1$ such that

$$\|\vec{x}_0 Q^{(t)}\|_n \leq cq^t \|\vec{x}_0\|_n \quad (29)$$

for any n -norm, where $\vec{x}_0 = [1 \ 0 \ \dots \ 0]$. Fix $\|\cdot\|_1$ in (29) as in Definition 4. Then, $\|\vec{x}_0 Q^{(t)}\|_1 = \sum_j |q_{1j}^{(t)}| = \sum_j q_{1j}^{(t)} = \vec{x}_0 Q^{(t)} \mathbf{1}$ and $\|\vec{x}_0\|_1 = \|[1 \ 0 \ \dots \ 0]\|_1 = 1$. Hence, for all sufficiently large t , from (29), we have

$$\vec{x}_0 Q^{(t)} \mathbf{1} = \sum_{j=1}^N q_j^{(t)} \leq cq^t = c \exp(-\gamma t) \quad (30)$$

where $\gamma = -\ln q > 0$. Note that (30) holds good for any trajectory \mathcal{B} and the constants c, q are independent of \mathcal{B} . Hence, from (23) and (25), we are done. \square

Through space/time quantization, Theorem 2 can also be applied to any other continuous-time, continuous-space models (e.g., random direction model (RDM), etc) provided that both the time it takes for a node to walk from one cell to another and the pause time are bounded. For any continuous-space analogue of our model, note that whenever nodes A and B are in the same cell, they are ‘in-range’ (the maximum distance between A and B is no larger than d). Note also that even when they meet in the continuous-space model, they may not in the quantized discrete-space model. For example, nodes A and B in Figure 2 are within a distance of d , but they are in different cells, i.e., they do not meet in the discrete-space model. In consequence, the inter-meeting time in a continuous-space model is upper-bounded by that in the discrete-space (quantized) model. Hence, Theorem 2 readily shows that the tail of the inter-meeting time of a continuous-space model also decays at least exponentially fast.

4. FROM EXPONENTIAL TO POWER-LAW INTER-MEETING TIME

4.1 Finite Boundary and Exponential Tail

So far we have proven that the inter-meeting time of two independent mobile nodes with mobility models such as RWP, RWM, and RDM in a bounded domain has at most exponential tails. These models are clearly different in defining each node’s mobility pattern. In fact, for other mobility models in a bounded domain (e.g., Brownian motion on a sphere, i.i.d. mobility model on a square, etc.), we can also show similar exponential behavior of the inter-meeting time. This observation along with our theoretical results assert that the exponential tail of the inter-meeting time seems a universal property and the empirically observed power-law decay of the inter-meeting time will not arise by simply tweaking the aforementioned mobility models.

What is common in all these mobility models? The finite boundary. Suppose that two mobile nodes (without pause) have not met for a long time (e.g., several hours). If there is no boundary, intuitively, it is more likely that they are moving toward different directions, hence chances are that they will not meet for the next several hours and the inter-meeting time is prolonged. In other words, the inter-meeting time has very strong memory and may result in a power-law type distribution.

However, when there is a finite boundary, after a long time (much longer than a typical amount of time for a node to hit the boundary), it forces the mobile nodes to bounce back once being hit, which tends to reset the nodes’ location and ‘erase’ the memory in the inter-meeting time. This nearly memoryless behavior caused by the existence of the finite boundary tends to shorten the inter-meeting time and actually results in an asymptotic exponential tail.

REMARK 7. While Theorems 1 and 2 in Section 3 clearly indicate that finite boundaries have significant impact on the tail behavior of the inter-meeting time distribution, we point out that they both require mobile nodes to be independent and the moment generating function of the pause time to be finite. (See Remark 3 and 5.) In other words, in the presence of non-negligible dependency among mobile nodes (e.g., they belong to the same group following similar paths all the time or totally different groups whose typical pathways do not overlap) or significant amount of pause time (e.g., pause time following power-law with infinite mean), the inter-meeting time could be power-law type even under a finite boundary. Another exception could be the case where there exists a considerable amount of correlations in the trajectory of a mobile node (e.g., the Gauss-Markov mobility model [24]). The effect of finite boundary on the inter-meeting time distribution under such spatial-temporal correlations and possibly power-law pause time is beyond the scope of this paper and left for future study.

4.2 Infinite Domain and Power-law Tail

In what follows, we will show through a class of simple isotropic random walks in an open-space without boundary (i.e., $\Omega = \mathbb{R}^2$) that the power-law decay of the inter-meeting time will arise by simply removing the boundary. We consider only a discrete-time model here, but similar results also hold for its continuous-time analogue, which we omit due to the space constraint.

In a two-dimensional (2-D) discrete-time isotropic random walk model, at the beginning of each time slot, the node chooses a random direction uniformly from $[0, 2\pi]$, travels for a random length R which is chosen from $(0, \infty)$ following certain distribution, then the process repeats itself. Denote by R_k the length of the k^{th} step and θ_k the random angle uniformly distributed over $[0, 2\pi]$. Then, the position of node A at time t ($t = 1, 2, \dots$) can be written as

$$A(t) = \sum_{k=1}^t \phi^A(k) = \sum_{k=1}^t R_k^A \exp(i\theta_k^A) \in \mathbb{R}^2, \quad (31)$$

where $A(0) = \mathbf{0}$. Since both sequences $\{R_k^A\}$ and $\{\theta_k^A\}$ are *i.i.d.* and independent from each other, $\phi^A(k) = R_k^A \exp(i\theta_k^A)$ ($k = 1, 2, \dots$) are also *i.i.d.* Thus, $A(t)$ is a sum of *i.i.d.* vectors, i.e., a random walk in 2-D. (Similarly for node B.)

The following result will be used in our proof of the main result in this section.

THEOREM 3. [Sparre-Andersen (S-A) Theorem in [1, 2, 29]]: For any one-dimensional discrete time random walk process starting at $x_0 \neq 0$ with each step chosen from a continuous, symmetric but otherwise arbitrary distribution, the First Passage Time Density (FPTD) to the origin asymptotically decays as $\sim t^{-3/2}$ with the number of steps t .

REMARK 8. The only assumption required in S-A Theorem is that each step of the random walker is chosen from a continuous (the probability of choosing a specific step length is zero) and symmetric distribution (at each step, the random walker goes left or right with equal probability). Denote the first passage time (FPT) as T_F , then by S-A Theorem, the probability density function of T_F decays as $\sim t^{-3/2}$, which means that the complementary cumulative distribution function (ccdf) of T_F decays as $\mathbb{P}\{T_F > t\} \sim t^{-1/2}$. The result is applicable for any initial position of the node except the origin.

Now we present our main result on the inter-meeting time of two isotropic random walkers.

THEOREM 4. Suppose that two independent nodes A and B move according to the 2-D isotropic random walk model described above. Then, there exists constant $C > 0$ such that the inter-meeting time T_I of nodes A and B satisfies

$$\mathbb{P}\{T_I > t\} \geq Ct^{-1/2}, \text{ for all sufficiently large } t. \quad (32)$$

PROOF. Define by $C(t) = A(t) - B(t) \in \mathbb{R}^2$ the difference vector between the position of node A and B at time t . Assume $\|C(0)\| = d$ and $\|C(1)\| > d$. Then, the inter-meeting time T_I becomes $T_I := \inf_{t>0} \{t : \|C(t)\| \leq d\}$.

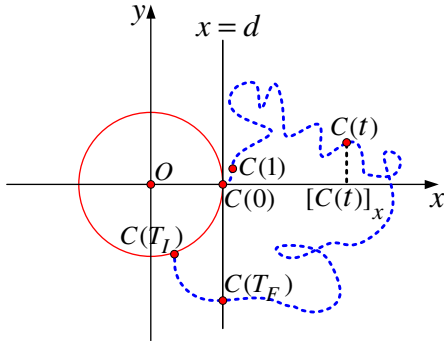


Figure 3: T_F is a lower bound on the inter-meeting time T_I as $C(t)$ must cross the line $x = d$ before crossing the circle of radius d .

Draw an x -axis connecting $C(0)$ and the origin as shown in Figure 3 and also y -axis accordingly. Let $[C(t)]_x$ be the projection of $C(t)$ onto the x -axis. Then, clearly, $[C(0)]_x = d$ and $[C(1)]_x > d$.⁶ Define by T_F the FPT of $[C(t)]_x$ to $x = d$ in 1-D (the vertical line tangent to the circle at $(d, 0)$). Suppose that the two nodes meet for the first time at $t = T_I$ since $t = 0$, or equivalently, $C(t)$ crosses the circle for the first time at $t = T_I$ after $t = 0$. Then, it is clear that $C(t)$ must have crossed the vertical line $x = d$ before it crosses the circle. In other words, we have $T_I \geq T_F$, and hence

$$\mathbb{P}\{T_I > t\} \geq \mathbb{P}\{T_F > t\}, \text{ for all } t > 0. \quad (33)$$

Observe that

$$C(t) = \sum_{k=1}^t \phi^A(k) - \sum_{k=1}^t \phi^B(k) = \sum_{k=1}^t \left(R_k^A e^{i\theta_k^A} - R_k^B e^{i\theta_k^B} \right),$$

Thus, we have

$$[C(t)]_x = \sum_{k=1}^t \left(R_k^A \cos \theta_k^A - R_k^B \cos \theta_k^B \right)$$

⁶This is always possible by making one time step small enough such that when the node gets out of the circle at $t = 1$, it is located outside that tangential line at $x = d$.

where R_k^A, R_k^B are all *i.i.d.* and so are θ_k^A, θ_k^B . In other words, $[C(t)]_x$ is one-dimensional random walk (sum of *i.i.d.* random variables) with each step distributed as $R_x = R^A \cos \theta^A - R^B \cos \theta^B$. Clearly, $R_x \stackrel{d}{=} -R_x$, so it is symmetric. Also, the distribution of R_x is continuous because the uniform distribution is continuous. Hence, by Theorem 3 and Remark 8,

$$\mathbb{P}\{T_F > t\} \sim t^{-1/2}.$$

In view of (33), this completes the proof. \square

The result in (32) is close to the empirical result reported in [17] that the coefficient of the power-law tail is nearly 0.4 (see Figure 4 in [17]). Recall that in Section 3, mobility models are analyzed through an irreducible Markov Chain (MC) by quantizing the space and time. Whenever there is a finite boundary, since the communication range d of two nodes is non-zero, we can always divide the system (bounded area) into a set of finite cells, each with diameter d . Hence, the MC has finite states. This finiteness of the state-space along with the irreducibility condition implies that the MC is positive recurrent [4]. In other words, starting from any initial position, the MC will return to a given state infinitely often, and the mean return time is finite. Further, Assumptions 3 and 4 guarantee that the MC will visit any arbitrary moving set of states also infinitely often and the mean return time to this moving set is finite. In fact, our result in Theorem 2 shows not only the mean of this return time (to a moving set) is finite, but its distribution is at most exponential, making all the moments also finite.

In contrast, however, when the node walks in an open space without boundary, the corresponding MC has an infinite number of states. As a result, even if the chain is recurrent, it may take infinitely long on average to return to a given state (null recurrence), which clearly rules out the possibility of an exponential tail of the inter-meeting time. In some sense, different recurrence properties in a bounded/unbounded space provide a quick explanation on the completely distinct tail behaviors of the inter-meeting time in these two situations.

5. SCALING THE SIZE OF THE SPACE

From Sections 3 and 4, we see that the finite boundary plays a key role in generating the exponentially decaying inter-meeting time or the FPT, and by removing the boundary, the power-law inter-meeting time distribution can be observed. However, the question is: *How to decide whether a domain is virtually bounded or not?* In fact, it is hard to believe that this change happens abruptly only at the infinity. If we set the boundary for RWM in Section 3 large enough such that the mobile node rarely hits the boundary under the timescale of interest, it is almost equivalent to the random walk in an open-space or we can say there is practically no boundary. In this regard, the question is: where does the transition from exponential to power-law happen? Or, equivalently, how do we choose (or scale) the size of the space (boundary) to observe the power-law inter-meeting time distribution over a given timescale?

The answer lies in the interaction between the timescale under discussion and the size of the boundary. As an example, consider a 2-D random walk model in (31) (or equivalently we can take 2-D Brownian motion in a continuous time case). Note that $\mathbb{E}\{A(t)\} = 0$ for all t from symmetry.

However, observe that

$$\mathbb{E}\{|A(t)|^2\} = \sum_{k,l=1}^t \mathbb{E}\{R_k R_l e^{i(\theta_k - \theta_l)}\} = \sum_{k=1}^t \mathbb{E}\{R_k^2\} = \sigma^2 t$$

from $\mathbb{E}\{e^{i\theta}\} = 0$ and the independence of the random variables for $k \neq l$. Thus, its standard deviation or the *average displacement* of node A during time t scales as $O(\sqrt{t})$.⁷ In other words, after a duration of time t , a node will typically travel a distance of $O(\sqrt{t})$ from its original position.

Suppose that the maximum timescale of interest is t_0 , i.e., we are interested in system dynamics only over $[0, t_0]$, and the ‘radius’ of the bounded space is r_0 . When $r_0 \ll \sqrt{t_0}$, the node will hit the boundary (thus ‘feel’ it) many times and a mobility model with a finite boundary is in effect. Hence all the results in Section 3 naturally apply. In contrast, when $r_0 \gg \sqrt{t_0}$, the node will almost never hit the boundary and we can say that the node is moving around practically in an unbounded space. Hence in this case results similar to those in Section 4 apply.

In order to grasp better idea of the space-time dynamics, let us take an example of the so-called *i.i.d.* mobility model and study different ways of scaling the size of the boundary as a function of time and their impact on the distribution of the inter-meeting time. In the *i.i.d.* mobility model, a node moves in a $D \times D$ square consisting of $N = D^2$ unit cells. At the start of each time slot, the node jumps to one of N cells with equal probability independently of everything else. Note that this is equivalent to setting $p_{ij} = 1/N$ in the RWM in Section 3.2. The two nodes ‘meet’ whenever they are in the same cell, and at any given time, this happens with probability $1/N = 1/D^2$. Suppose now that the length of the boundary is a function of time, i.e., $D = D(t)$. When $D(t)$ grows over time (i.e., $D(t) \rightarrow \infty$ as $t \rightarrow \infty$), note that

$$\begin{aligned} \mathbb{P}\{T_I > t\} &= [1 - 1/D^2(t)]^t \\ &= [1 - 1/D^2(t)]^{D^2(t) \cdot \frac{t}{D^2(t)}} \sim e^{-g(t,N)}, \end{aligned} \quad (34)$$

where $g(t, N) \triangleq t/D^2(t)$. Hence, the scaling function $g(t, N)$ completely determines the tail behavior of the inter-meeting time. For example, when $g(t, N) \sim \beta t$ or $D(t) = O(1)$, we have $\mathbb{P}\{T_I > t\} \sim e^{-\beta t}$ as expected. When $g(t, N) \sim \alpha \log t$, we get $\mathbb{P}\{T_I > t\} \sim e^{-\alpha \log t} = t^{-\alpha}$. In other words, we get the desired power-law behavior when the length of the boundary is scaled as $D(t) \sim \sqrt{t/(\alpha \log t)}$. This is also consistent with the aforementioned discussion that the average displacement scales as $O(\sqrt{t})$. By maintaining the size of the space as $D(t) = O(\sqrt{t/(\alpha \log t)})$ or larger, we can ensure that the ‘world’ is expanding at about the same rate (or larger) as the node, so the node rarely feels the existence of the boundary. If $D(t) \ll O(\sqrt{t/(\alpha \log t)})$, the size of the world grows much slower than the average displacement of a node, so the node inevitably hits the boundary many times and the non-power-law or exponential behavior of the inter-meeting time starts to kick in.

This first-order scaling rule via $g(t, N)$ also provides a way to choose a ‘right’ size of the world in the random walk model for the study of MANET. For instance, if one is interested in some performance metric that makes sense only

⁷We implicitly assume that $\sigma^2 = \mathbb{E}\{R^2\} < \infty$. If $\mathbb{E}\{R^2\} = \infty$, then under constant velocity, the mobile node tends to ‘spread out’ quicker and its average displacement will grow faster than $O(\sqrt{t})$.

over a certain time period, e.g., hours, days, or even weeks (could be possible in delay-tolerant network settings), then the scaling function $g(t, N)$ tells us approximately how to set the size of the boundary in which the MANET is operating so as to make the inter-meeting time power-law distributed. Another possible way is to dynamically increase the size of the boundary $D(t)$ as time goes on to control the ‘frequency’ of hitting the boundary as desired.

6. SIMULATION

In this section we provide simulation results to support our theoretical results in Sections 3 and 4, as well as the discussion on scaling the size of the space in Section 5. We here consider RWP and random walk model (RWM) with/without boundary. In any case, the speed of a node is chosen uniformly from 1 m/sec to 1.68 m/sec with a mean value of 1.34 m/sec [22], since this is known as the mean walking speed of human beings. The transmission range is set to 50 meters, and we have square domain for all mobility models with finite boundary in our simulations.

As mentioned before, the interaction between the timescale under discussion and the size of the boundary is essential in determining the tail behavior of the inter-meeting time. To see this interaction, we fix the simulation time period and change the size of the boundary using RWP and RWM (including both bounded and essentially unbounded domain cases) models to observe possibly different tail behaviors of the inter-meeting time.

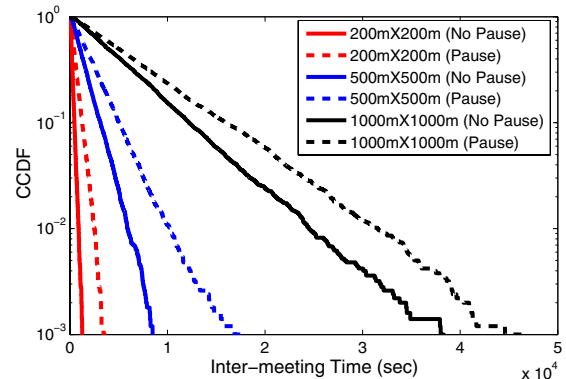
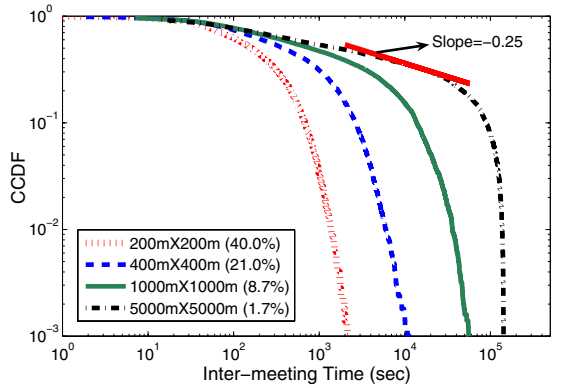


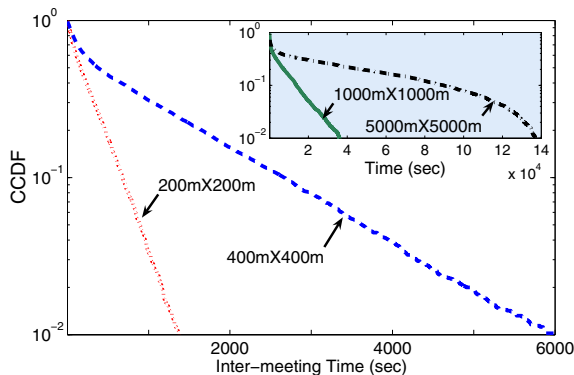
Figure 4: CCDF of the inter-meeting time for RWP models under different sizes of the domain: ‘Pause’ means an exponentially distributed pause time with a mean value of 3 minutes, and ‘No pause’ means zero pause time. Figure is drawn on a semi-log scale. In any case, the slope increases for larger size of the domain.

Figure 4 shows the complementary distribution (CCDF) of the inter-meeting time $\mathbb{P}\{T_I > t\}$ on a semi-log scale for RWP models under varying sizes of the domain. We set the simulation period as 40 hours,⁸ and simulate zero pause time case (‘no pause’) as well as an exponentially distributed pause time with mean of 3 minutes (‘pause’). Note that no matter how large the boundary is, the inter-meeting time always has an exponential tail, which is in good

⁸The timescale which matters in real network is commonly in the order of hours (less than one day). Hence, we choose 40 hours, which is long enough to observe the accurate tail behavior of the inter-meeting time in this timescale.



(a) Log-Log scale



(b) Semi-Log scale

Figure 5: CCDF of the inter-meeting time for RWM models under different sizes of the domain on a log-log scale (a) and on a semi-log scale (b). The hitting frequency to the boundary decreases as the size of the domain increases. For 5000×5000 square meters size, a node rarely hits the boundary and thus moves like in an unbounded space and produces power-law behavior of the inter-meeting time, as seen in (a). Exponential behavior of the inter-meeting time distribution under smaller domains (e.g., $200m \times 200m$, $400m \times 400m$) is clearly shown in (b) (straight line on a semi-log scale). Power-law type behavior starts to appear in mid to large domain size as shown in the inset in the top right of (b).

agreement with our theoretical results in Proposition 1 and Theorem 1. Recall from Section 5 that in case of *i.i.d.* mobility models, the slope of the exponential tail is given by $g(t, N)/t = 1/D^2$. In other words, the slope of the tail in a semi-log scale is inversely proportional to the area of the bounded domain (D^2). Similarly, even for RWP models, we notice from Figure 4 that the product of the slope of the tail and the area of the domain (ranging from 200×200 up to 1000×1000 square meters) remains almost constant, which implies that the intuition obtained from *i.i.d.* mobility models can also be applied to RWP models.

Figure 5 shows CCDF of the inter-meeting time on a log-log scale for random walk models (RWM) under different sizes of the domain, where the boundary is always reflective. To observe the transition of inter-meeting time behavior from an exponential type to a power-law more clearly, we draw the simulation results both on a log-log scale in Figure 5(a) and on a semi-log scale in Figure 5(b). In this case,

we set the simulation period to 40 hours.⁹ Here, we use a very simple and widely used random walk model where a node changes directions uniformly from $[0, 2\pi]$ every 50 seconds [7]. For each size of the domain simulated, we calculate the hitting frequency defined as the ratio between the number of steps within which the node hits the boundary and the total number of steps. For example, when the size of domain is 200×200 , the hitting frequency is quite high (40.0%), thus the inter-meeting time behaves like an exponential (The CCDF of inter-meeting time is a straight line on semi-log scale, as Figure 5(b) shows.) as expected from our theoretical results. In contrast, when the size of the domain is very large (5000×5000 square meters), the hitting frequency is as low as 1.7%, which means that it is practically an unbounded domain for the duration of the timescale (40 hours). Hence, as Theorem 4 shows, the inter-meeting time follows a power-law distribution. (The CCDF of inter-meeting time is a straight line on log-log scale, as Figure 5(a) shows.) Further, the slope of this CCDF on a log-log scale is shown to be around 0.25, which is also consistent with our findings in Section 4. In addition, for a large domain size (e.g., $5000m \times 5000m$), the power-law behavior is dominant over up to $O(10^4)$ seconds, followed by a sharp decrease beyond that timescale. This behavior is also consistent with the measurement studies in [17].

7. CONCLUSION

In this paper we have proven that one of the fundamental reasons, for the discrepancy on the tail behavior of inter-meeting time between the recent empirical data and the theoretical/simulation results based on most of the current mobility models, is the *finite boundary*. We have also shown that simply removing the boundary can quickly change the inter-meeting time distribution from exponential to power-law by studying a simple random walk in an open space. Further, our guidelines on scaling the size of the domain also help better understand the true role of the boundary. Our theoretical results and findings provide guideline on mobility modeling, performance analysis, and protocol design, to survive the ‘curse’ of the power-law distribution of the inter-meeting time in MANET.

APPENDIX

Proof of Lemma 1: In view of Proposition 2 in [25] and $\|P_{k_1} \cdots P_{k_M}\|_\infty < 1$ for any index set $\{k_1, \dots, k_M\}$ ($k_i \in \{1, \dots, N\}$, $i \in \{1, \dots, M\}$) and $M < \infty$, the following condition is satisfied by $A(\mathcal{I}, k)$:

$$\lim_{k \rightarrow \infty} A(\mathcal{I}, k) = \mathbf{0}, \text{ for all } \mathcal{I} = \{n_1, n_2, \dots\} \quad (35)$$

where $n_l \in \{1, 2, \dots, M\}$ for all $l \in \mathbb{N}$.

Define a discrete-time system whose state at time t is given by a $1 \times N$ vector $\vec{X}(t)$ where $\vec{X}(t) = \vec{X}(t-1)P_{n_t} = \cdots = \vec{X}(0)A(\mathcal{I}, t) = \vec{x}_0 A(\mathcal{I}, t)$. Relation (35) means that the system is absolutely asymptotic stable (Section 3.1.1 in [36]), which is equivalent to saying that the system is absolutely exponentially stable (Definition 3.1 in [36]), i.e., for any \vec{x}_0 ,

⁹Since $\sqrt{144000} * 1.35 \approx 512$ (144000 seconds are 40 hours, 1.35 is the square root of the second moment of step length), we expect that the two smallest domains can be looked as bounded, whereas the other two can be looked as essentially unbounded from our discussion in Section 5.

there exists constant $c > 0$, $0 < q < 1$ such that for all sufficiently large k ,

$$\|\vec{x}_0(\mathcal{I}, k)\| \leq cq^k \|\vec{x}_0\|, \quad (36)$$

where $\|\cdot\|$ is any vector norm. Specifically, (36) is true for n -norms $\|\cdot\|_n$.

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