Distributed Spectrum Management and Relay Selection in Interference-limited Cooperative Wireless Networks

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Outline

Introduction

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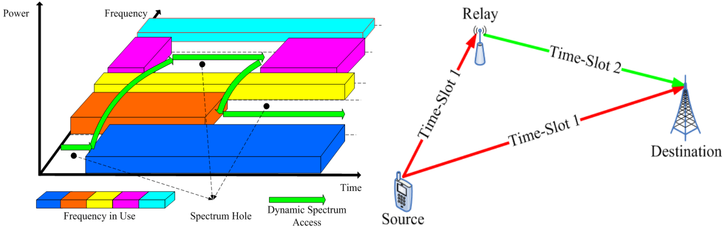
- Emerging multimedia services require high data rate
- Need to maximize transport capacity of wireless networks

Introduction



- Increase transport capacity by leveraging frequency and spatial diversity
  - Dynamic spectrum access: improve spectral efficiency (frequency diversity)
  - Cooperative communications: enhance link connectivity (spatial diversity)

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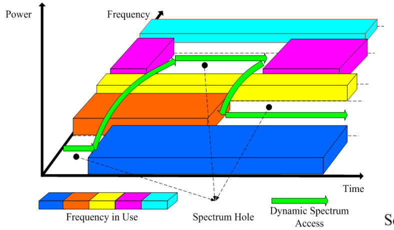

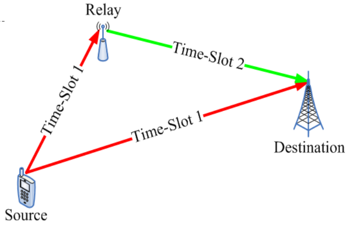

- Increase transport capacity by leveraging frequency and spatial diversity
  - Dynamic spectrum access: improve spectral efficiency (frequency diversity)
  - Cooperative communications: enhance link connectivity (spatial diversity)
- Open challenge: Distributed control strategies
  - to dynamically jointly assign portions of spectrum and cooperative relays
  - to maximize network-wide data rate
  - in interference-limited infrastructure-less networks

Related Work – Leveraging Spectral And Spatial Diversity

- Centralized control in interference-free networks
  - Y. Shi, S. Sharma, Y. T. Hou, and S. Kompella, “Optimal relay assignment for cooperative communications,” in *Proc. ACM Intl. Symp. on Mobile Ad Hoc Networking and Computing (MobiHoc)*, HK, China, May 2008.

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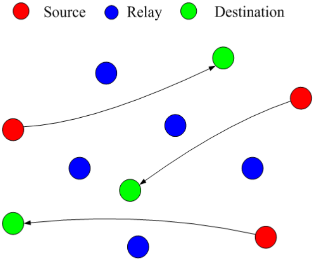
- We focus on distributed control in interference-limited infrastructure-less networks

System Model



- Interference-limited infrastructure-less cooperative network
  - Uncoordinated source-destination pairs
  - Each source transmits using direct link or through cooperative relaying
  - Dynamically access a portion of spectrum to avoid interference
- Assumptions
  - Single hop (no layer-3 routing)
  - Each source uses at most one relay
  - Each relay can be used by at most one source

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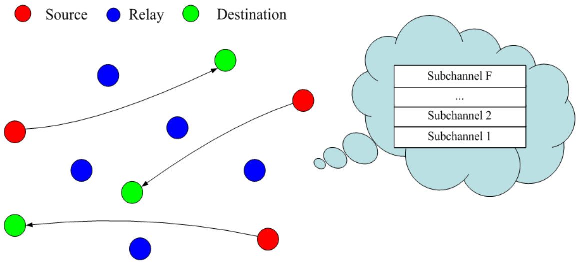

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Problem Formulation – Overall Model

Objective

Maximize sum utility (capacity, log-capacity) of multiple concurrent traffic sessions

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- Relay selection (whether to cooperate or not, and through which relay)
- Dynamic spectrum access (which channel(s) to transmit on, and at what power)

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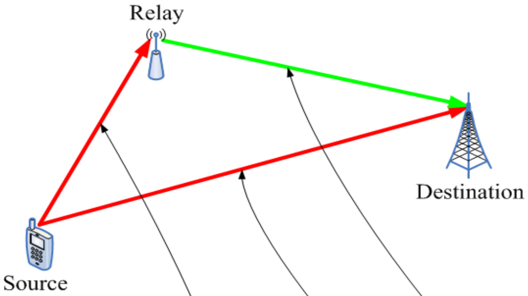
Subject to

- Total power constraint
- Relay selection constraint

Problem Formulation – Link Capacity Model



Cooperative Transmission (Decode-and-Forward) [1]

$$C_{cop}^{s,r,f} = \frac{B_f}{2} \min(\log_2(1 + SINR_{s2r}^{s,r,f}), \log_2(1 + SINR_{s2d}^{s,s,f} + SINR_{r2d}^{r,s,f}))$$

– Choices of relay node and transmit power are important!

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Direct Transmission

$$C_{dir}^{s,f} = B \log_2 \left(1 + SINR_{s2d}^{s,s,f}\right)$$

– Capacity of cooperative transmission may be higher or lower than that of direct transmission. Cooperate or not?

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Problem Formulation – Mixed Integer Non-Convex Problem

$$\underset{P,\ Q,\ \alpha}{\text{Maximize}} \quad U = \sum_{s \in \mathcal{S}} U_s(P,\ Q,\ \alpha) \rightarrow \text{Utility function}:\ \log(C_s)$$

$$\text{Subject to} \quad \alpha_r^s \in \{0,\ 1\},\ \forall s \in \mathcal{S},\ \forall r \in \mathcal{R} \rightarrow \text{Integer},\ 1: \text{selected},\ 0: \text{not}$$

$$\sum_{r \in \mathcal{R}} \alpha_r^s \leq 1,\ \forall s \in \mathcal{S} \rightarrow \text{Each session uses at most one relay}$$

$$\sum_{s \in \mathcal{S}} \alpha_r^s \leq 1,\ \forall r \in \mathcal{R} \rightarrow \text{Each relay selected by at most one session}$$

$$P_s^f \geq 0, \forall s \in \mathcal{S},\ \forall f \in \mathcal{F} \rightarrow \text{Power allocation for source},\ \text{real},\ \text{nonnegative}$$

$$Q_r^f \geq 0, \forall r \in \mathcal{R},\ \forall f \in \mathcal{F} \rightarrow \text{Power allocation for relay},\ \text{real},\ \text{nonnegative}$$

$$\sum_{f \in \mathcal{F}} P_s^f \leq P_{max}^s,\ \forall s \in \mathcal{S} \rightarrow \text{Power budget of source}$$

$$\sum_{f \in \mathcal{F}} Q_r^f \leq Q_{max}^r,\ \forall r \in \mathcal{R} \rightarrow \text{Power budget for relay}$$

- Link capacity C_s is function of SINR
- SINR is nonlinear and non-convex with respect to $P,\ Q$ and α

Proposed Solution Algorithm

MINCoP

- NP-HARD in general

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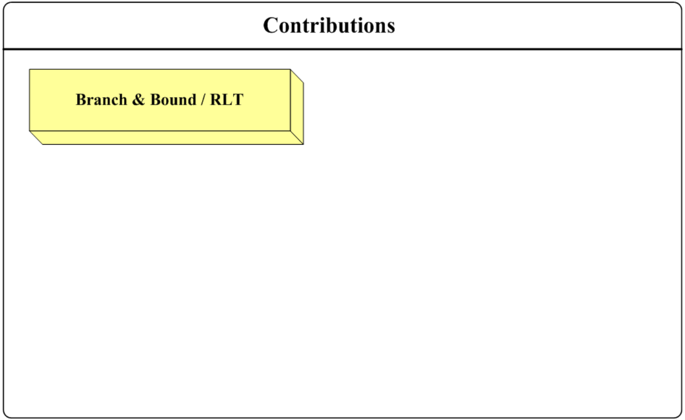
Contributions

Proposed Solution Algorithm

MINCoP

- NP-HARD in general



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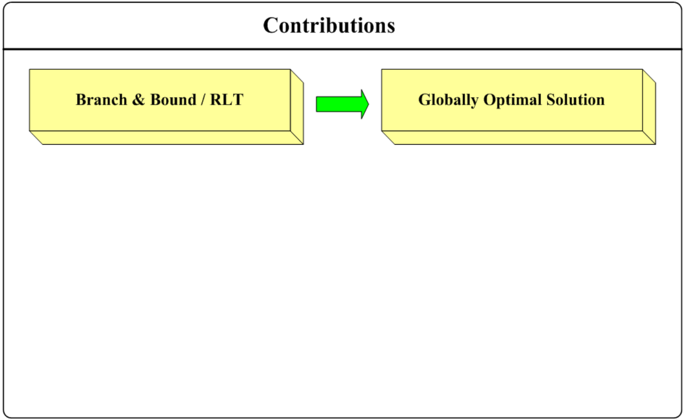

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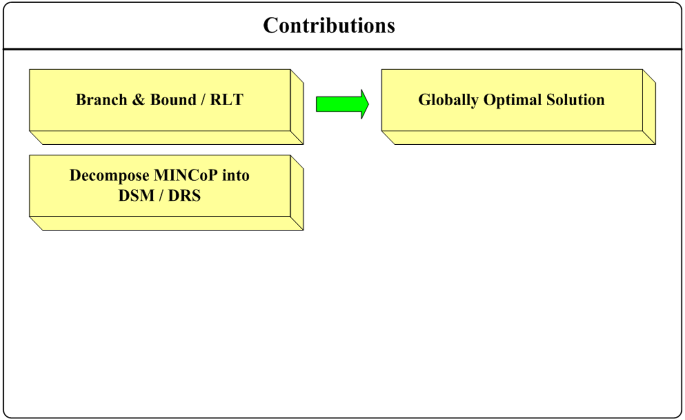

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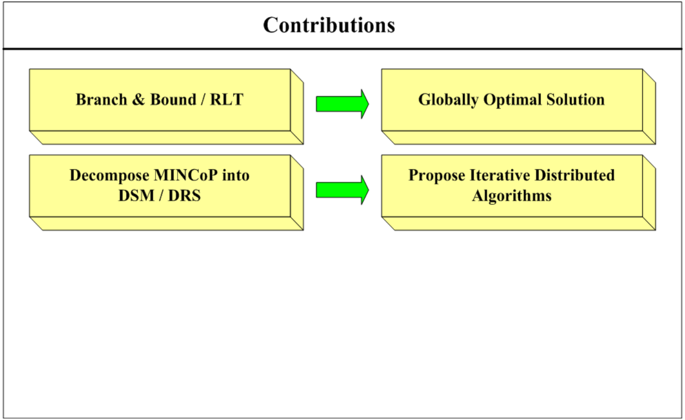

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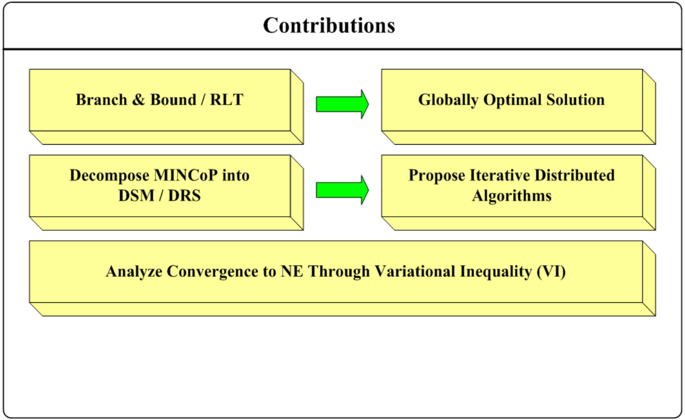

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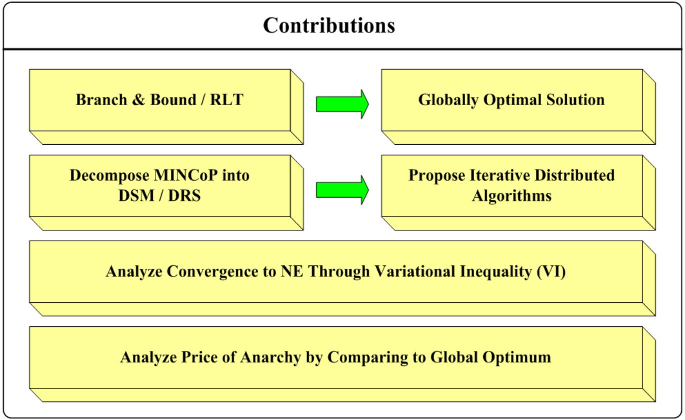
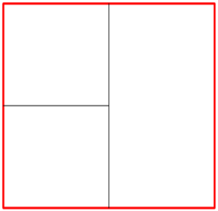
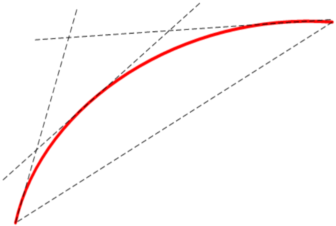


Globally Optimal Algorithm

Central Idea

- Based on a combination of branch-and-bound (B&B) and convex relaxation.
  - B&B: Iteratively partition the original MINCoP problem into a series of subproblems
  - Convex Relaxation: Relax each subproblem to be convex





Globally Optimal Algorithm – Basic Steps

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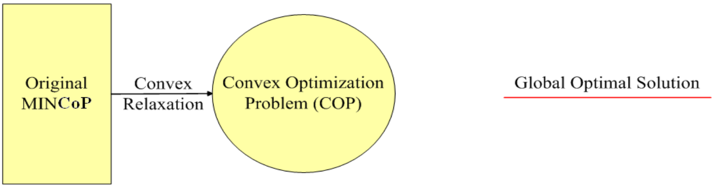

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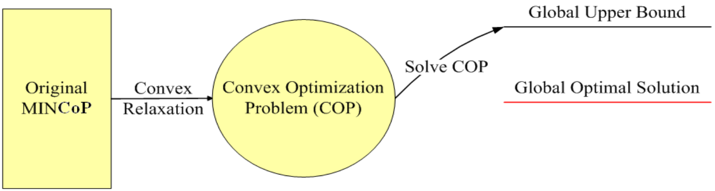

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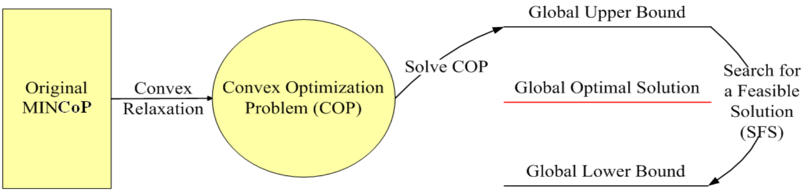

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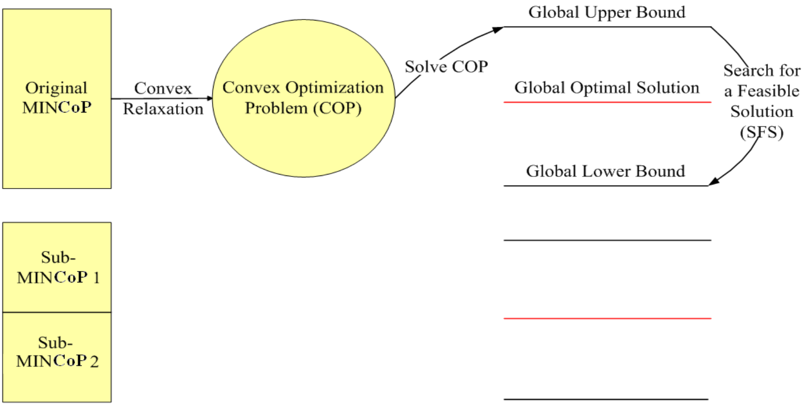

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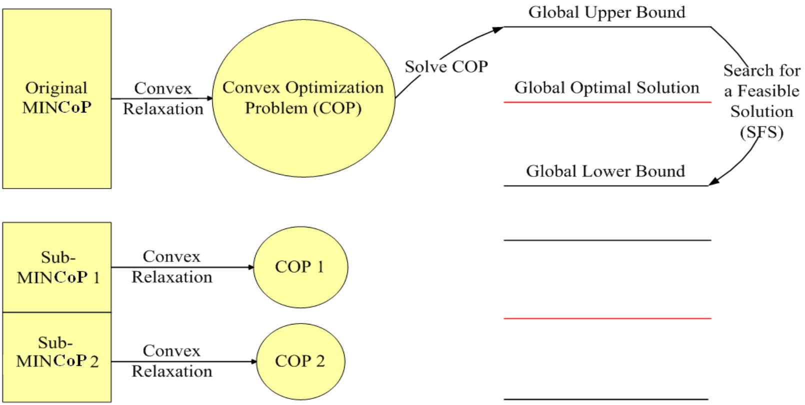

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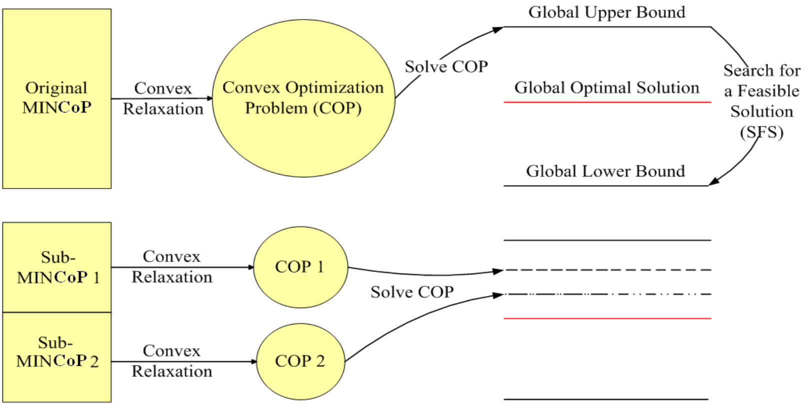

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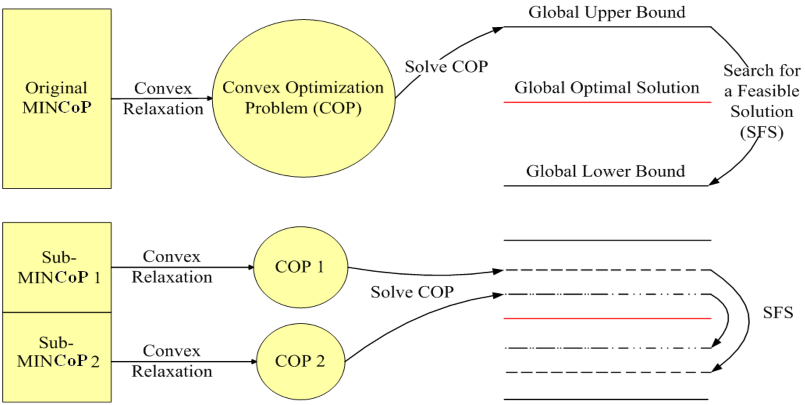

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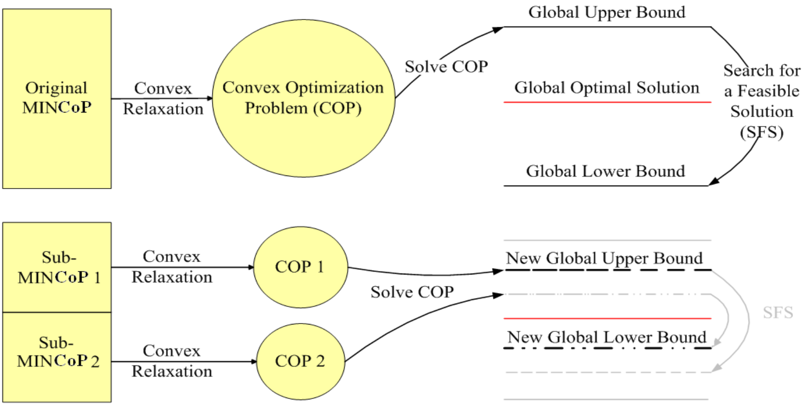

Example: Reduction of Feasible Set

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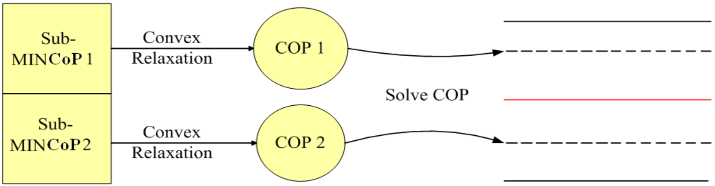

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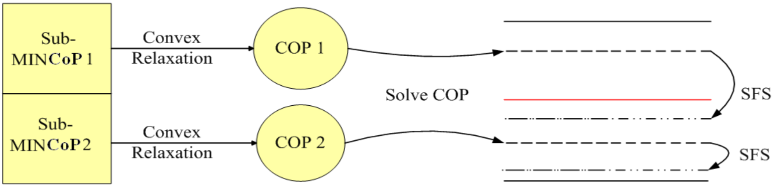

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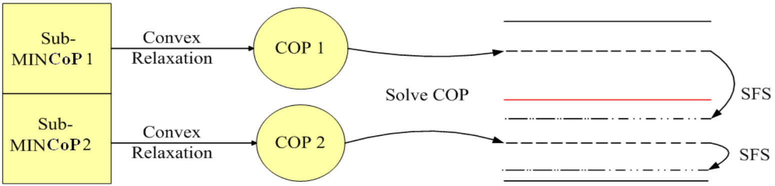

Upper bound of Sub-MINCoP 2 is smaller than lower bound of Sub-MINCoP 1 – Sub-MINCoP 2 pruned

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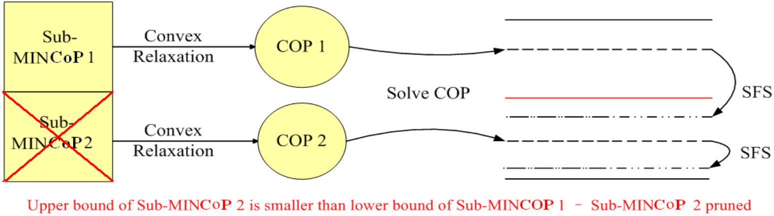

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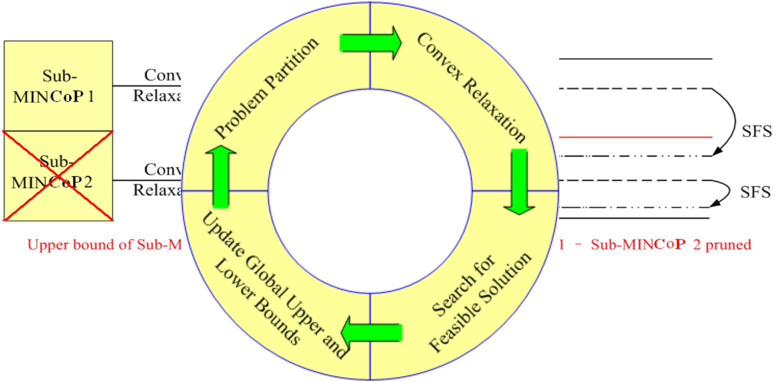
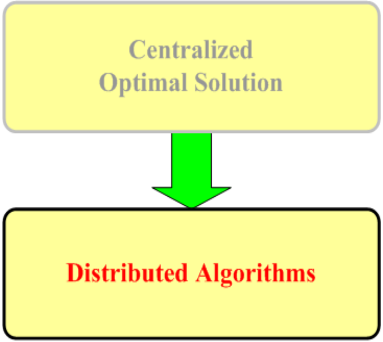


Proposed Solution Algorithm – Distributed Through VI

VI to facilitate theoretical analysis

- Hard to obtain global optimum in distributed way
- Design algorithms to achieve Nash Equilibrium
- Nash Equilibrium analysis is challenging due to complicated expression of utility functions
- Variational Inequality Theory [2]
  - Broader applicability than classical game theory results
  - Well developed tools for existence and convergence analysis
  - Applies to our problem under certain conditions

[2] Gesualdo Scutari, Daniel P. Palomar, Francisco Facchinei, and Jong-Shi Pang, “Convex Optimization, Game Theory, and Variational Inequality Theory in Multiuser Communication Systems,” IEEE Signal Processing Magazine, vol. 27, no. 3, pp. 35-49, May 2010.


Centralized
Optimal Solution
Distributed Algorithms

Proposed Solution Algorithm – Nash Equilibrium & VI

Nash Equilibrium

- Concept from noncooperative game theory
- At Nash Equilibrium no user has incentive to deviate from current transmission strategy
- x_i: Transmission strategy of player i
- x_{-i}: Transmission strategy of all other players except i
- Nash Equilibrium problem is defined to find x^* such that

$$\boldsymbol{x_i^*} = \arg\max_{\boldsymbol{x_i} \in \mathcal{Q}_i} \boldsymbol{f_i}(\boldsymbol{x_i}, \boldsymbol{x_{-i}^*}), \ \forall \boldsymbol{i}$$

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Variational Inequality (VI)

- Generalization of optimization and game theory

$$(\boldsymbol{x} - \boldsymbol{x}^*)^{\mathrm{T}}\mathrm{F}(\mathrm{x}^*) \geq \mathbf{0}, \ \forall \boldsymbol{x} \in \mathcal{X}$$

F: Vector of gradient functions of utility function

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F: Vector of gradient functions of utility function

- Each solution of VI is a Nash Equilibrium

Proposed Solution Algorithm – Challenges With VI

Monotonicity

- VI theory requires mapping function F to be at least component-wise monotonic

$$\boldsymbol{F} = (\nabla_{\boldsymbol{x}_s} \boldsymbol{U}_s)_{s \in \mathcal{S}} \rightarrow \textbf{Vector of gradient of utility function}$$

- Hard for simultaneous optimization of spectrum allocation and relay selection

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- Utility function is not smooth due to $\min(\cdot)$ operation when DF is used

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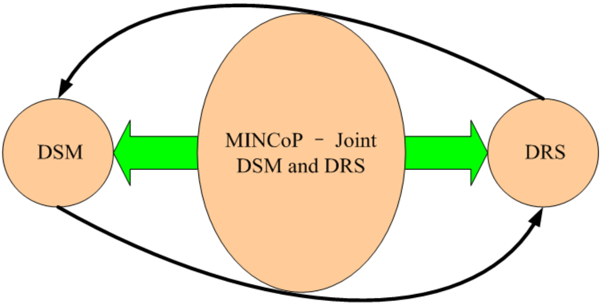
Decomposability

- Relay selection variables are coupled with each other
- Domain set of a player is function of transmission strategy of other players, hence not fixed
- Resulting Nash Equilibrium problem or VI problem is more complicated

Proposed Solution Algorithm – Monotonicity

- Decompose original problem into two subproblems
- Design distributed algorithm for each subproblem
- Perform two algorithms iteratively
- Monotonicity condition is easily satisfied by each subproblem



Proposed Solution Algorithm – Differentiability

Non-smooth Function

$$C_{cop}^{s,r,f} = \min(C_{s2r}^{s,r,f}, C_{sr2d}^{s,r,f})$$

Approximation Function

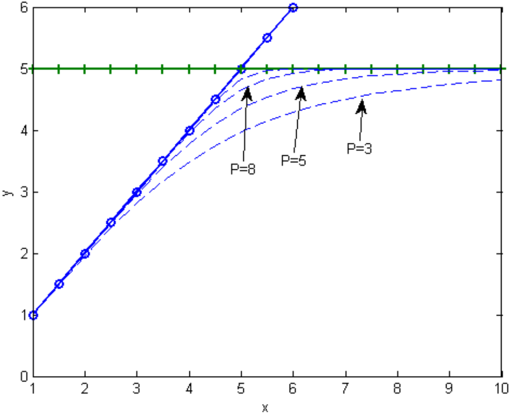
- Approximate $\min(\cdot,\cdot)$ based on ℓ_p-norm function

$$\widehat{C}_{cop}^{s,r,f} = \ell_P^{-1}((C_{s2r}^{s,r,f})^{-1}, (C_{sr2d}^{s,r,f})^{-1}) = \left\{ \left[\left(\frac{1}{C_{s2r}^{s,r,f}} \right)^P + \left(\frac{1}{C_{sr2d}^{s,r,f}} \right)^P \right]^{\frac{1}{P}} \right\}^{-1}$$

Lemmas

- Approximation function is continuously differentiable
- Approximation function is concave when SINR is not too low

Proposed Solution Algorithm – Differentiability



Approximation function (smooth) can approximate the original min (non-smooth) with arbitrary precision

Proposed Algorithm – Convergence of DSM Algorithm

Lemma

- Game of DSM can be reformulated as a VI problem $\mathrm{VI}(\mathcal{X}, F)$

$$U_s(x_s,\ x_{-s}) = \log(C_s(x_s,\ x_{-s})) \rightarrow \text{Utility function}$$

$$F = (\nabla_{x_s} U_s)_{s \in \mathcal{S}} \rightarrow \text{Vector of gradient of utility function}$$

$$\mathcal{X} = \prod_{s \in \mathcal{S}} \mathcal{X}_s \rightarrow \text{Cartesian product of domain sets}$$

- There exists at least one solution for $\mathrm{VI}(\mathcal{X}, F)$ (also a Nash Equilibrium)

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Theorem

If any two sessions are located sufficiently far away from each other, then a Gauss-Seidel scheme based on the local best response converges to a VI solution, therefore to a Nash Equilibrium.

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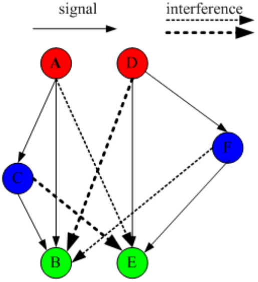
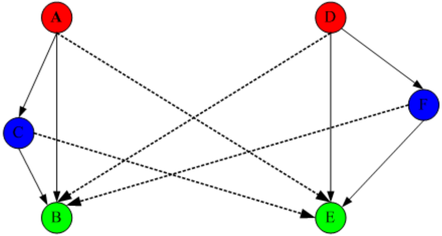
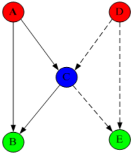


Do not converge - Strong Interference (much coupling)



Converge - Weak Interference (less coupling)

Proposed Algorithm – Decomposability



- Dynamic relay selection (DRS) as a game
  - Each session is a player $\rightarrow$ Maximize $U_s(\alpha_s, \alpha_{-s})$
  - Relay selection variables are coupled with each other

$$\sum_{s \in \mathcal{S}} \alpha_r^s \leq 1, \ \forall r \in \mathcal{R}$$

  - Resulting joint domain set cannot be decomposed as Cartesian product of multiple sub-domains
- Nash Equilibrium problem with coupled domain sets is called Generalized Nash Equilibrium problem (GNE)

Lemma

Resulting GNE can be reformulated as a VI, called QVI, and there exists at least one VI solution which is also a Nash Equilibrium solution.

Proposed Solution Algorithm – DRS

Theorem

The following penalized iterative algorithm converges to a VI solution, which is also a Nash Equilibrium solution [3].

$$\widehat{U}_s(\alpha_s,\ \alpha_{-s}) = U_s(\alpha_s,\ \alpha_{-s}) \underbrace{-\frac{1}{2\rho_k}\sum_{r\in\mathcal{R}}\left(\max\left(0, u_k^r + \rho_k\left(\sum_{s\in\mathcal{S}}\alpha_r^s - 1\right)\right)\right)^2}_{\text{Penalization}}$$

$$\rho_{k+1} = \rho_k + \Delta\rho,$$

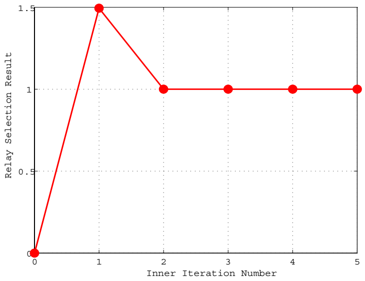
$$u_{k+1}^r = \max\left(0,\ u_k^r + \rho_k\left(\sum_{s\in\mathcal{S}}\alpha_{s,r}^k - 1\right)\right)$$

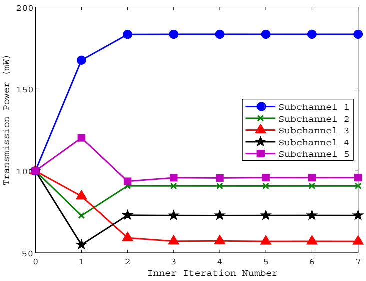
[3] Jong-Shi Pang and Masao Fukushima, “Quasi-variational inequalities, generalized Nash equilibria, and multi-leader-follower games,” Computational Management Science, 2(1):21-56, Jan. 2005.

Performance Analysis – System Setup

- System Parameters
  - A terrain of 1500 $\mathrm{m} \times$ 1500 m
  - Session number: 2, 3, 5, 10, 10
  - Relay number: 10, 5, 5, 5, 5
  - Channel number: 4, 5, 5, 5, 2
  - Channel gain: $G_{mn} = d^{-\gamma}(m, n)$
  - Path loss factor: $\gamma = 4$
  - Average AWGN noise power: 10^{-7} mW

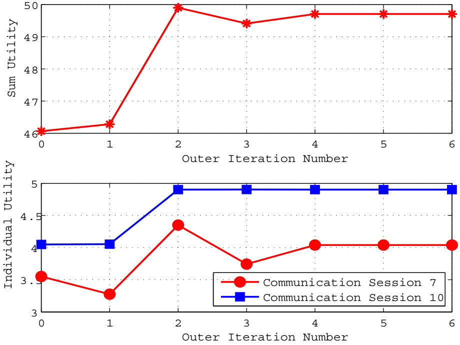
Performance Analysis – Convergence of DRS and DSM





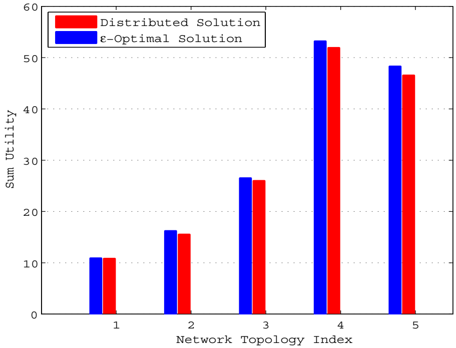
- Both DRS and DSM converge fast

Performance Analysis – Convergence of Joint DSM and DRS



- Iteration of DRS and DSM converges fast in practice

Performance Analysis – Price of Anarchy



- $\varepsilon = 95\%$: Centralized algorithm achieves at least 95% of the global optimum
- Distributed algorithm can achieve a performance close to the optimum within several percentages

Conclusions

- Conclusions
  - Formulation of joint dynamic spectrum allocation and relay selection in interference-limited infrastructure-less networks

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  - Designed distributed algorithms by decomposing MINCoP in two subproblems

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  - Formulation of joint dynamic spectrum allocation and relay selection in interference-limited infrastructure-less networks
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  - Formulation of joint dynamic spectrum allocation and relay selection in interference-limited infrastructure-less networks
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  - Demonstrated existence of Nash Equilibrium, convergence to Nash Equilibrium through VI for each subproblem
  - Performance close to the optimum achieved by distributed algorithm
- Future Work: Implement distributed algorithm in USRP2/GNU Radio testbed

Thanks for your attention

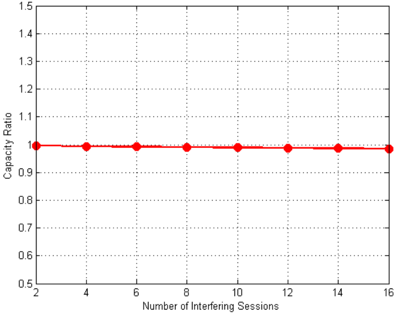
Problem Formulation – Interference Model

- Interference depends on power allocation, relay selection, network scheduling
- Average-based model is used for tractability

$$I = \frac{1}{2}(I_{time\_slot\_1} + I_{time\_slot\_2})$$

- Experiment verified its negligible impact on the overall network performance

Problem Formulation – Interference Model



- Comparison between the average-based interference model and exact interference in synchronization-based cooperative network
- Capacity ratio: $\frac{C_{avg}}{C_{rea}}$
- C_{avg}: Capacity calculated using the average-based interference model
- C_{rea}: Capacity in reality
- The average-based approximation approximates reality very well

Proof of Convergence of DSM

- Domain set $\mathcal{X}$ is closed and convex
- Mapping function F_s is strongly monotonic
- "Sufficiently far away" is a sufficient condition
- Every session uses a relay, session s uses relay node r
- Gradient vector of session s with respect to x_s

$$J_{x_s}(U_s) = \left(\left(\frac{\partial U_s}{\partial P_s^f} \right)_{f=1}^{F}, \left(\frac{\partial U_s}{\partial Q_r^f} \right)_{f=1}^{F} \right)$$

- Define a matrix $[\gamma]_{ij}$ as

$$[\gamma]_{sg} \triangleq \begin{cases} \alpha_s^{\min}, & \text{if } s = g, \\ -\beta_{sg}^{\max}, & \text{otherwise,} \end{cases}$$

- $\alpha_s^{\min} \triangleq \inf_{x \in \mathcal{X}} \lambda_{\text{least}}(J_{x_s x_s}(U_s))$ and $\beta_{sg}^{\max} \triangleq \sup_{x \in \mathcal{X}} \|J_{x_g x_s}(U_g)\|$
- $\lambda_{\text{least}}(A)$ is the eigenvalue of A with the smallest absolute value
- Sufficient far away implies that $[\gamma]_{sg}$ is a P-matrix

Proof of Convergence of DRS

- Sufficient to show
  - DRS converges
  - Every accumulation point corresponds to a VI solution
- According to Theorem 3 in [3], $\max\left(0, u_k^r + \rho_k\left(\sum_{s\in\mathcal{S}}\alpha_r^s - 1\right)\right)$ is bounded
- Penalization item tends to zero as ρ_k tends to infinity
- According to Theorem 3 in [3], every accumulation point corresponds to a VI solution

[3] Jong-Shi Pang and Masao Fukushima, "Quasi-variational inequalities, generalized Nash equilibria, and multi-leader-follower games," Computational Management Science, 2(1):21-56, Jan. 2005.

Proof of Concavity

- Approximation function $\widehat{C}_{cop}^{s,r,f}$ is monotonically increasing
- Domain set $\mathcal{X}$ in the VI problem $\mathrm{VI}(\mathcal{X}, F)$ is bounded
- Only need to show $\widehat{C}_{cop}^{s,r,f}$ is a concave function
- A function is concave if it is concave when restricted to any line in the domain

Proposed Solution Algorithm – Practical Issues

- Application Scenarios
  - Multiple co-existing pre-established source-destinations
  - Independent set of transmissions with primary interference constraints
- Dynamic spectrum access
  - SINR measurement is needed at destination and relay nodes, or
  - at source node via control information
  - Cooperative MAC protocol is desired, e.g., CoCogMAC [4]
- Dynamic relay selection
  - Relay periodically broadcasts a “price” frame to claim its price

[4] L. Ding, T. Melodia, S. N. Batalama, and J. D. Matyjas, “Distributed Routing, Relay Selection, and Spectrum Allocation in Cognitive and Cooperative Ad Hoc Networks,” in *Proc. IEEE Communications Society Conference on Sensor, Mesh and Ad Hoc Communications and Networks (SECON)*, Boston, USA, Jun. 2010.