

# Optimal Gateway Selection in Multi-domain Wireless Networks: A Potential Game Perspective

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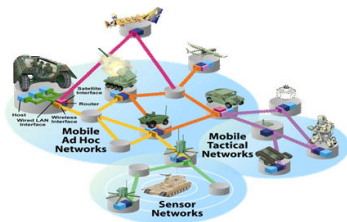
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# Overview

- 1 Motivation
- 2 Gateway Selection Game
- 3 Equilibrium Selective Learning
- 4 Performance Evaluation
- 5 Conclusions

# Coalition Networks with Multiple Domains



## Scenario:

- Coalition networks with **heterogenous** groups.
- Inter-connected via wireless links, e.g., IEEE 802.11, WiMAX, UAV, satellite, 3G/4G etc.

## Example:

- Joint military missions, US-UK
- Disaster rescue teams, fire-fighters and police officers
- Wireless sensor networks of different organizations, e.g., Internet of Things (IoT), Smart Planet Solutions

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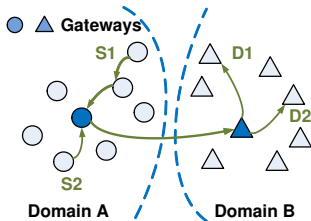
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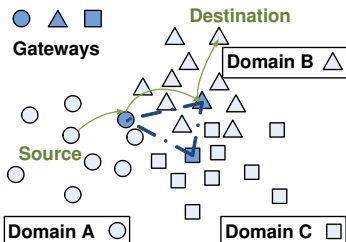
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## Solution:

Designate **gateway** nodes



# Cost Efficient Gateway Selection

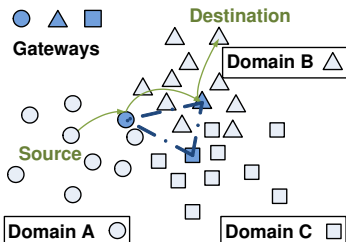


Each pair of nodes has a **cost**, e.g., routing metric cost, such as

- hop count, RIP, AODV etc.
- Euclidean distance
- ETX, ETT, RTT
- Energy consumption etc.



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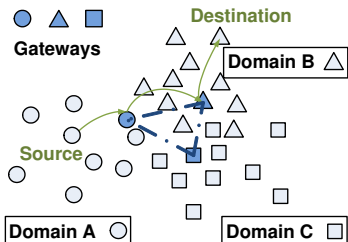
For a single domain

**Intra-domain cost**

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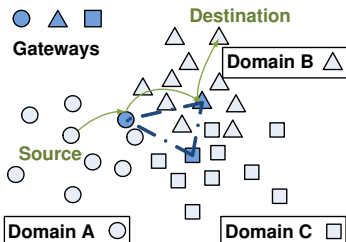
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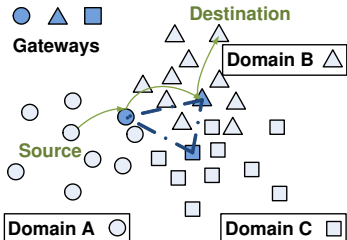
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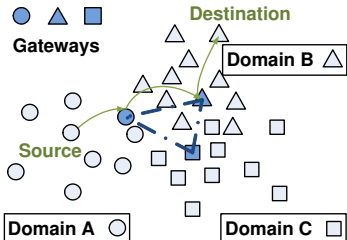
## Question:

How to select the set of gateways s.t. the **overall** cost is minimized?

# Challenges

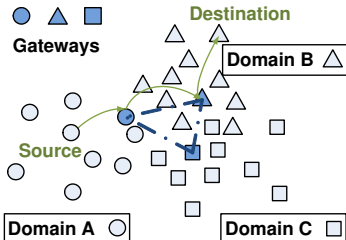


# Challenges



- Combinatorial nature of solution space

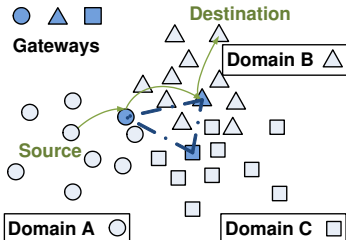
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- Combinatorial nature of solution space

Distributed solution

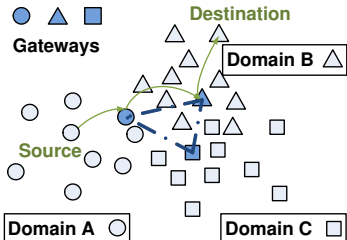
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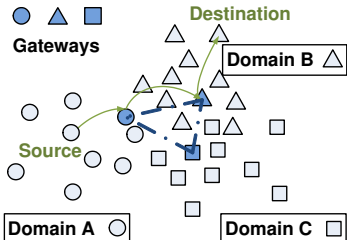
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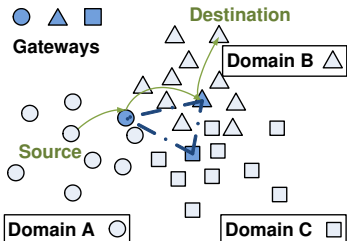


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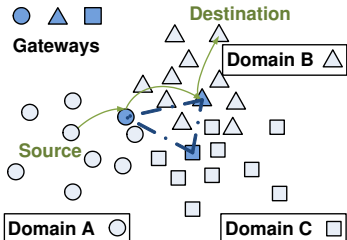
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Local information only

potential game theory & equilibrium selective learning

# Network Model

- $\mathcal{M}$  : the set of domains in the coalition network
- $\mathcal{N}_m$ : the set of nodes in the domain
- $g_m^i = 1$ : node  $i$  is selected as the gateway node and  $g_m^i = 0$  o.w. and  $\hat{i}_m = \operatorname{argmax}_{i \in \mathcal{N}_m} g_m^i$  be the selected gateway node
- $\mathbf{g}_m = \{g_m^1, g_m^2, \dots, g_m^{|\mathcal{N}_m|}\}$ : the **gateway selection strategy** of domain  $m$
- $\mathbf{s} = \{\mathbf{g}_1, \mathbf{g}_2, \dots, \mathbf{g}_{|\mathcal{M}|}\}$ : the joint **gateway selection profile** of the network
- Satellite/UAV/3G/4G link: cost  $\eta$  (expensive), to enforce always-on connectivity
- A pair of node  $i$  and  $j$ :  $c(i, j) \geq 0$  is the associated symmetric link cost,  $c(i, j) = \eta$  if out of range
- $c'(i, j) \triangleq \min(c(i, j), \eta)$

# Gateway Selection Game

For each single domain

Minimize (**Local** information and observation only)

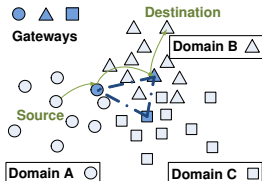
$$\mathcal{U}_m(\mathbf{g}_m, \mathbf{g}_{-m}) = \sum_{i \neq \hat{i}_m, i \in \mathcal{N}_m} c(i, \hat{i}_m) + \sum_{n \neq m, n \in \mathcal{M}} c'(\hat{i}_m, \hat{i}_n) \quad (1)$$

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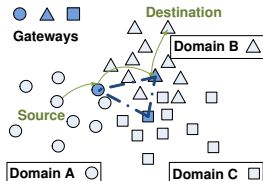
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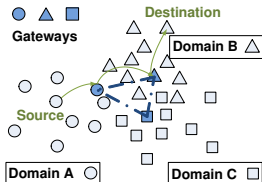
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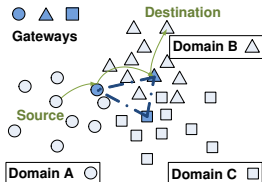


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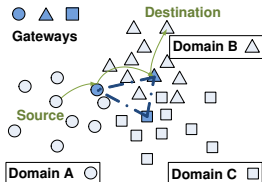
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## For overall network

Minimize (**intra-domain cost** + **cost of backbone communication links**)

$$\mathcal{R}(\mathbf{s}) = \sum_m \sum_{i \neq \hat{i}_m, i \in \mathcal{N}_m} c(i, \hat{i}_m) + \sum_{(\hat{i}_m, \hat{i}_n) \in \text{MCG}(\mathbf{s})} c'(\hat{i}_m, \hat{i}_n). \quad (2)$$

# Existence of Nash Equilibrium

## Theorem

*The gateway selection game has a Nash equilibrium, which minimizes, either locally or globally, the following function*

$$\mathcal{F}(\mathbf{s}) = \sum_m \sum_{i \neq \hat{i}_m, i \in \mathcal{N}_m} c(i, \hat{i}_m) + \sum_{(\hat{i}_m, \hat{i}_n) \in \text{CCG}(\mathbf{s})} c'(\hat{i}_m, \hat{i}_n). \quad (3)$$

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- Nash equilibrium may not be unique
- Multiple Nash equilibria have different performance
- To capture the (in)efficiency of Nash equilibrium, **Price of Anarchy** and **Price of Stability** are introduced

$$\text{Price of Stability} = \frac{\text{value of best equilibrium}}{\text{value of optimal solution}}$$

## Efficiency of Nash Equilibria

For  $|\mathcal{M}| = 2$

For two player gateway selection games, the best Nash Equilibrium is the global network optimum solution, i.e., the price of stability is 1.

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**All else**

If the triangle inequality does not hold, the price of stability of an  $|\mathcal{M}|$ -player gateway selection game is at most  $(1 + \delta)$ , where

$$\delta = \frac{\eta \left( \frac{|\mathcal{M}|}{2} + \frac{1}{|\mathcal{M}|} - \frac{3}{2} \right)}{\min_{m \in \mathcal{M}} \min_{\mathbf{g}_m} \sum_{i \neq \hat{i}_m(\mathbf{g}_m), i \in \mathcal{N}_m} c \left( i, \hat{i}_m(\mathbf{g}_m) \right)}. \quad (4)$$



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$$\begin{aligned} & \Pr(\mathbf{g}_m(t+1) = \widetilde{\mathbf{g}}_m) \\ &= \frac{\exp^{-\mathcal{U}_m(\widetilde{\mathbf{g}}_m, \mathbf{g}_{-m}(t))/\tau}}{\exp^{-\mathcal{U}_m(\widetilde{\mathbf{g}}_m, \mathbf{g}_{-m}(t))/\tau} + \exp^{-\mathcal{U}_m(\mathbf{g}_m(t), \mathbf{g}_{-m}(t))/\tau}} \end{aligned} \quad (5)$$

and

$$\Pr(\mathbf{g}_m(t+1) = \mathbf{g}_m(t)) = 1 - \Pr(\mathbf{g}_m(t+1) = \widetilde{\mathbf{g}}_m) \quad (6)$$

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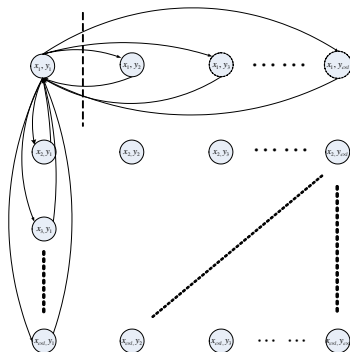
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where  $\tau$  is a small positive constant, a.k.a., the **smoothing factor** of the algorithm.

- It is known that as  $\tau \rightarrow 0$ , B-logit converges to the best Nash equilibrium with arbitrarily high probability.

# Proof (sketch)



- Note  $\Pr(s' \rightarrow s'')$

$$\frac{1}{|\mathcal{M}|} \frac{1}{|\mathcal{N}_m|} \frac{\exp^{-\mathcal{U}(s'')/\tau}}{\exp^{-\mathcal{U}_m(\tilde{\mathbf{g}}_m, \mathbf{g}_{-m}(t))/\tau} + \exp^{-\mathcal{U}_m(\mathbf{g}_m(t), \tilde{\mathbf{g}}_{-m}(t))/\tau}}$$

- Verify

$$\pi(s') = \frac{\exp^{-\mathcal{F}(s')/\tau}}{\sum_{s \in \mathcal{S}} \exp^{-\mathcal{F}(s)/\tau}}$$

satisfies the **detailed balance equation**, i.e.,  
 $\pi(s') \Pr(s' \rightarrow s'') = \pi(s'') \Pr(s'' \rightarrow s')$

- B-logit algorithm induces a reversible, irreducible, and aperiodic Markov chain and it is the **unique** steady state distribution.
- By taking  $\tau \rightarrow 0$ , we have

$$\pi(\mathbf{s}^*) \rightarrow 1, \text{ where } \mathbf{s}^* = \arg\min_{s \in \mathcal{S}} \mathcal{F}(s)$$

# Generalization of B-logit

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## $\gamma$ -logit algorithm family ( $\Gamma$ ):

- $\gamma$ -logit shares the same structure as B-logit except in (5), where the probability is calculated as

$$\Pr(\mathbf{g}_m(t+1) = \widetilde{\mathbf{g}}_m) = \frac{\exp^{-\mathcal{U}_m(\widetilde{\mathbf{g}}_m, \mathbf{g}_{-m}(t))/\tau}}{\gamma(\mathbf{s}', \mathbf{s}'')} \quad (7)$$

where  $\mathbf{s}' = \{\mathbf{g}_m(t), \mathbf{g}_{-m}(t)\}$  and  $\mathbf{s}'' = \{\widetilde{\mathbf{g}}_m, \mathbf{g}_{-m}(t)\}$  are two gateway selection profiles in  $\mathcal{S}$ , and  $\gamma$  satisfies

### 1 Symmetry

$$\gamma(\mathbf{s}', \mathbf{s}'') = \gamma(\mathbf{s}'', \mathbf{s}'), \forall \mathbf{s}' \in \mathcal{S}, \mathbf{s}'' \in \mathcal{S},$$

### 2 Feasibility

$$\gamma(\mathbf{s}', \mathbf{s}'') \geq \max\left(\exp^{-\mathcal{U}_m(\mathbf{s}')/\tau}, \exp^{-\mathcal{U}_m(\mathbf{s}'')/\tau}\right).$$

- B-logit is a **special case** of  $\gamma$ -logit algorithm with

$$\gamma(\mathbf{s}', \mathbf{s}'') = \gamma(\mathbf{s}'', \mathbf{s}') = \exp^{-\mathcal{U}_m(\mathbf{s}')/\tau} + \exp^{-\mathcal{U}_m(\mathbf{s}'')/\tau}.$$

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- Which is **better**?
- Each  $\gamma$ -logit algorithm induces a Markov chain with different transition probability matrix, where

$$P_{i,j}(\gamma) \triangleq \Pr(\mathbf{s}^i \rightarrow \mathbf{s}^j) = \frac{1}{|\mathcal{M}|} \frac{1}{|\mathcal{N}_m|} \frac{\exp^{-\mathcal{U}(\mathbf{s}^j)/\tau}}{\gamma(\mathbf{s}^i, \mathbf{s}^j)}$$

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- The mixing rate of a Markov chain is determined by the **second largest eigenvalue modulus (SLEM)**, i.e.,

$$\mu(P(\gamma)) = \max(|\lambda_2(P(\gamma))|, |\lambda_{|S|}(P(\gamma))|).$$

- The **smaller**  $\mu(P(\gamma))$  is, the **faster**.

## Solution: MAX-logit Algorithm

### MAX-logit:

For every time slot  $t$ :

- Randomly select one of the players, say  $m$ , to update its gateway selection while other domains remain unchanged.
- Denote the current gateway selection of domain  $m$  as  $\mathbf{g}_m(t)$ . Domain  $m$  randomly selects a node in its domain as the gateway candidate. Denote the candidate gateway selection strategy by  $\widetilde{\mathbf{g}}_m$ . Domain  $m$  updates as

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Denote  $\mu^{MAX}$  as the second largest eigenvalue modulus associated with MAX-logit algorithm.

### Theorem

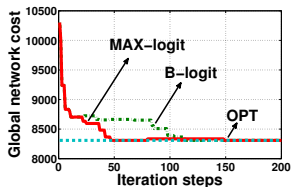
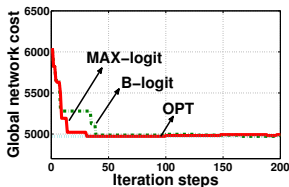
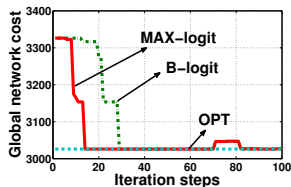
Denote  $\mu(P(\gamma))$  as the second largest eigenvalue modulus induced by an arbitrary  $\gamma$ -logit algorithm in  $\Gamma$ . We have  $\mu^{MAX} \leq \mu(P(\gamma))$ .

## Evaluation setup

- $|\mathcal{M}|$  domains where each domain has  $|\mathcal{N}|$  nodes
- For each domain, nodes are randomly deployed in a round area with radius  $125m$ , centered at a random point within the square field of  $1000 \times 1000m^2$
- **Link cost:**
  - 1 Euclidean distance: Network optimum solution is the best Nash ( $\gamma$ -logit algorithms converge to the network optimum solution)
  - 2 Random cost:  $\gamma$ -logit algorithm converges to the approximate  $1 + \delta$  solution (Nash equilibrium)
  - 3 Randomly select  $p\%$  of the links in the network and add random cost offset which is uniformly distributed between 0 and 5% of the original cost
- Global link cost  $\eta = 500$ ,  $|\mathcal{M}| = 2, 3, 4$
- $\tau = 0.0001$

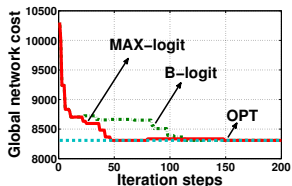
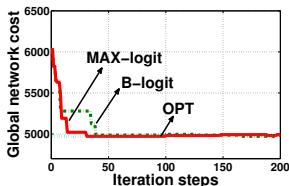
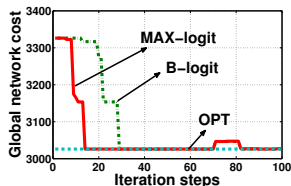
## Euclidean Distance Scenarios

- $p\% = 0\%$
- 2, 3, 4 domains where each domain has 20 nodes



## Euclidean Distance Scenarios

- $p\% = 0\%$
- 2, 3, 4 domains where each domain has 20 nodes



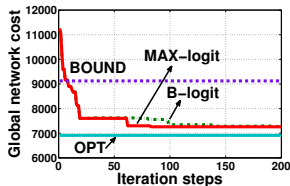
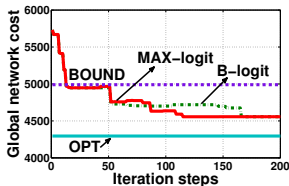
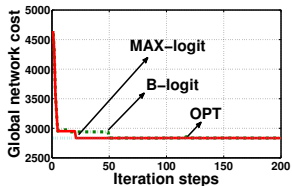
Nodes per domain	2 domains	3 domains	4 domains
5 nodes	16.06%	24.52%	33.85%
10 nodes	25.00%	29.81%	28.55%
20 nodes	11.96%	20.19%	20.36%
30 nodes	5.87%	16.46%	17.60%

- Average over 5000 sample runs
- Performance improvement declines when no. of nodes increases



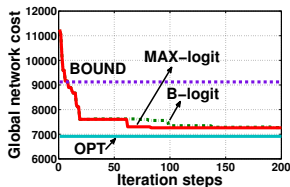
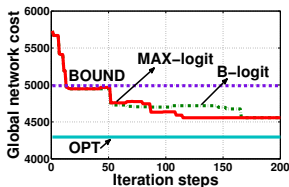
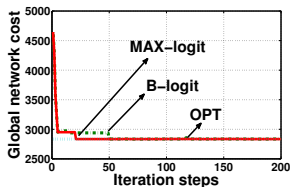
# Random Cost Scenarios

- $p = 50$ , i.e., 50% of the links in the network are associated with random link cost
- 2, 3, 4 domains where each domain has 20 nodes



# Random Cost Scenarios

- $p = 50$ , i.e., 50% of the links in the network are associated with random link cost
- 2, 3, 4 domains where each domain has 20 nodes



Nodes per domain	2 domains	3 domains	4 domains
5 nodes	21.84%	24.46%	27.38%
10 nodes	21.00%	21.44%	21.56%
20 nodes	9.54%	9.13%	5.47%
30 nodes	1.90%	1.93%	2.24%

Table: Convergence rate improvement by MAX-logit when  $p = 50$ .

# Conclusions

- Interactive gateway selection by multiple domains in coalition networks
- In a potential game framework, the existence and inefficiency of Nash equilibria are characterized (two domains, multi-domains)
- Equilibrium selective learning: generalized B-logit into  $\gamma$ -logit, or  $\Gamma$
- Propose MAX-logit which converges to the best Nash equilibrium at the fastest speed in  $\Gamma$
- Other applications of potential games in power control, channel allocation, spectrum sharing content distribution etc.