Temporal Reachability Graphs

John Whitbeck, Marcelo Dias de Amorim, Vania Conan and Jean-Loup Guillaume
Intro : Contact Traces

\[ t \]

\[ t + dt \]
Intro : Contact Traces
## Intro : Contact Traces

<table>
<thead>
<tr>
<th>Time</th>
<th>Node 1</th>
<th>Node 2</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>a</td>
<td>b</td>
<td>UP</td>
</tr>
<tr>
<td>0</td>
<td>d</td>
<td>e</td>
<td>UP</td>
</tr>
<tr>
<td>0</td>
<td>c</td>
<td>e</td>
<td>UP</td>
</tr>
<tr>
<td>1</td>
<td>a</td>
<td>b</td>
<td>DOWN</td>
</tr>
<tr>
<td>1</td>
<td>d</td>
<td>e</td>
<td>DOWN</td>
</tr>
<tr>
<td>1</td>
<td>c</td>
<td>e</td>
<td>DOWN</td>
</tr>
<tr>
<td>1</td>
<td>b</td>
<td>d</td>
<td>UP</td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>b</td>
<td>UP</td>
</tr>
</tbody>
</table>

Contact trace : symmetric single-hop information

Opportunistic routing : asymmetric multi-hop connectivity over time
**Intro : Contact Traces**

<table>
<thead>
<tr>
<th>Time</th>
<th>Node 1</th>
<th>Node 2</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>a</td>
<td>b</td>
<td>UP</td>
</tr>
<tr>
<td>0</td>
<td>d</td>
<td>e</td>
<td>UP</td>
</tr>
<tr>
<td>0</td>
<td>c</td>
<td>e</td>
<td>UP</td>
</tr>
<tr>
<td>1</td>
<td>a</td>
<td>b</td>
<td>DOWN</td>
</tr>
<tr>
<td>1</td>
<td>d</td>
<td>e</td>
<td>DOWN</td>
</tr>
<tr>
<td>1</td>
<td>c</td>
<td>e</td>
<td>DOWN</td>
</tr>
<tr>
<td>1</td>
<td>b</td>
<td>d</td>
<td>UP</td>
</tr>
<tr>
<td>2</td>
<td>a</td>
<td>b</td>
<td>UP</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>...</td>
</tr>
</tbody>
</table>

Contact trace : symmetric single-hop information
Opportunistic routing : asymmetric multi-hop connectivity over time
Outline

1. From time-varying connectivity graphs to reachability graphs
2. Efficient calculation of reachability graphs
3. Results: bounds on communication capabilities
From time-varying connectivity graphs to reachability graphs
Temporal reachability graphs

**TRG Definition**

In a \((\tau, \delta)\)-reachability graph, an arc exists from node \(A\) to \(B\) at time \(t\) if a space-time path exists from \(A\) to \(B\) leaving \(A\) at time \(t\) and arriving at \(B\) before \(t + \delta\) given that each single-hop takes time \(\tau\).
Temporal reachability graphs

TRG Definition

In a \((\tau, \delta)\)-reachability graph, an arc exists from node \(A\) to \(B\) at time \(t\) if a space-time path exists from \(A\) to \(B\) leaving \(A\) at time \(t\) and arriving at \(B\) before \(t + \delta\) given that each single-hop takes time \(\tau\).

<table>
<thead>
<tr>
<th>Delay Tolerance ((\delta))</th>
<th>TRG at (t = 0s)</th>
<th>TRG at (t = 1s)</th>
<th>TRG at (t = 2s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\delta = 1s)</td>
<td><img src="image1" alt="" /></td>
<td><img src="image2" alt="" /></td>
<td><img src="image3" alt="" /></td>
</tr>
<tr>
<td>(\delta = 2s)</td>
<td><img src="image4" alt="" /></td>
<td><img src="image5" alt="" /></td>
<td><img src="image6" alt="" /></td>
</tr>
</tbody>
</table>
Time-varying dominating set

TVDS Definition

A time-varying dominating set (TVDS) of a temporal reachability graph, is a time-varying set of nodes such at at all times $t$, the node in the TVDS are a regular dominating set of the directed reachability graph at time $t$. 
Why reachability graphs?

- On reachability graphs, certain routing performance questions become easy
  - Upper-bound on average delivery ratio at time $t$ (e.g., point-to-point, broadcast)
  - Size of the “temporal dominating set” at time $t$ (for offloading)
- New analysis angles on connectivity graphs
  - Asymmetric / Symmetric connectivity phases
  - Good receivers = large incoming node degree
  - Good senders = large outgoing node degree

The real challenge is calculating a reachability graph from a regular time-varying graph!
Why reachability graphs?

- On reachability graphs, certain routing performance questions become easy
  - Upper-bound on average delivery ratio at time $t$ (e.g., point-to-point, broadcast)
  - Size of the “temporal dominating set” at time $t$ (for offloading)
- New analysis angles on connectivity graphs
  - Asymmetric / Symmetric connectivity phases
  - Good receivers = large incoming node degree
  - Good senders = large outgoing node degree

The real challenge is calculating a reachability graph from a regular time-varying graph!
Efficient calculation of reachability graphs
“Adding” reachability graphs

\[ \begin{align*}
\delta & \oplus \mu = \delta + \mu \\
\end{align*} \]

\[
\begin{array}{cccc}
A & \rightarrow & B & \rightarrow & C \\
B & \rightarrow & C & \rightarrow & A \\
\end{array}
\]
“Adding” reachability graphs

\[ R_\delta \oplus R_\mu = R_{\delta+\mu} \]

\[ \begin{align*}
    \text{d=1 at } t & \quad \text{d=1 at } t+1 \\
    A & \quad A \\
    \quad B & \quad \quad B \\
    \quad \quad C & \quad \quad C
    \end{align*} \]
But not quite so easy...

<table>
<thead>
<tr>
<th>Messages are created out of sync with the contact trace’s granularity $(t \neq k\eta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Good upper (too many arcs) and lower (a few missed arcs) approximations</td>
</tr>
<tr>
<td>• Bounds are equal for all $t = k\eta$</td>
</tr>
</tbody>
</table>
But not quite so easy...

Example taken from the Rollernet trace with $\tau = 5s$ and $\delta = 1min$
But not quite so easy...

Graph can evolve faster than the transmission time ($\eta < \tau$)

- Composition over *families* of TRGs with close $\delta$ values
- Parallel computation of families
Results: bounds on communication capabilities
Example 1: Rollernet ($\tau = 5s$)

Proportion of connected pairs

Dominating set size

Time (min)

$\delta = 10s$

$\delta = 1min$

$\delta = 3min$
Example 2: Stanford High ($\tau = 1s$)

![Graph showing temporal reachability](image)

- **$\delta = 20\text{min}$**
- **$\delta = 1\text{h}$**
- **$\delta = 2\text{h}$**

Proportion of connected pairs and dominating set size over time (h).
Rollernet: Average density vs. maximum delay $\delta$ for different edge traversal times $\tau$
Rollernet: Average dominating set size vs. maximum delay $\delta$ for different values of $\tau$ (in seconds).
Conclusions

Contributions

- Formalization of temporal reachability graphs (TRG)
- Fast implementation of the conversion from regular time-varying graphs
- Powerful tool for analyzing performance bounds in opportunistic networks (e.g., asymmetry, max delivery ratio)
- Opens up many new perspectives (modeling, community detection)

Lessons for opportunistic networks

- Point to point communications with acceptable delays are very hard
- However usually possible to reach everyone in the network through a small dominating set (Offloading)
Thank You!

More info at: http://www-npa.lip6.fr/~whitbeck

Calculation & visualization code: http://github.com/neush/ditl