# Multihop wireless networks: capacity limits and how to approach them

Leandros Tassiulas
University of Thessaly, Volos, Greece
www.inf.uth.gr/~leandros

Mobihoc 2008 Keynote presentation

# The theme

- Theoretical framework for multi-hop wireless networks
- Dynamic model of information flow and topology variations
- Capture interactions across layers, from PHY to transport
- Quantify the notion of transport capacity as a design goal
- Specify capacity achieving policies
- Quantify other performance objectives, fairness, delay, energy consumption…
- Develop a systematic approach to the design of network control policies, based on system and optimization theory

References related to the presentation in: www.inf.uth.gr/~leandros

# Broader Perspective-Capacity Notions

- Shannon capacity – information theory

  The fundamental notion of communication capacity
- Key to achieve capacity in point-to-point links.
- Several results available in single-hop networks, i.e. broadcast and multiaccess channels
- Complex multihop networks defy information theoretic modeling and analysis
- Evidence to that: we hardly know anything even for the three-node two-hop relay channel

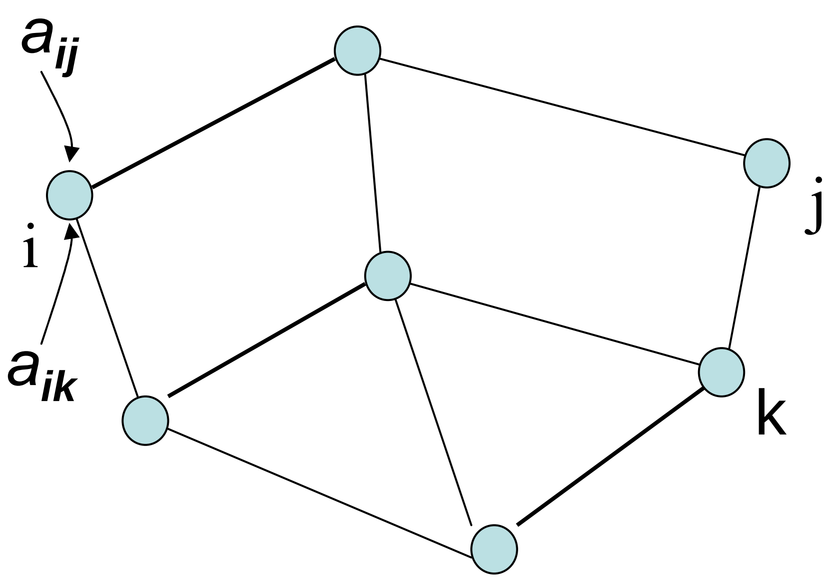
# Queuing theory-stochastic networks

- sufficient for understanding information transport at the network layer for segregated flows
- Very successful in traffic engineering
- Inadequate to capture cross layer interactions or non-traditional mixing of information streams

# Network "(Information) Theory(s)"

- Revived interest in 2000´s, towards developing a theoretical basis for communication networks
- Our dynamic system and optimization theory based approach, parallel and complementary to other current approaches: cooperative communications, network coding, capacity scaling laws, ...
- These recent advances motivated major initiatives i.e. ITmanet, CBmanet (US), Future Internet (EU), that fuel in return their further development

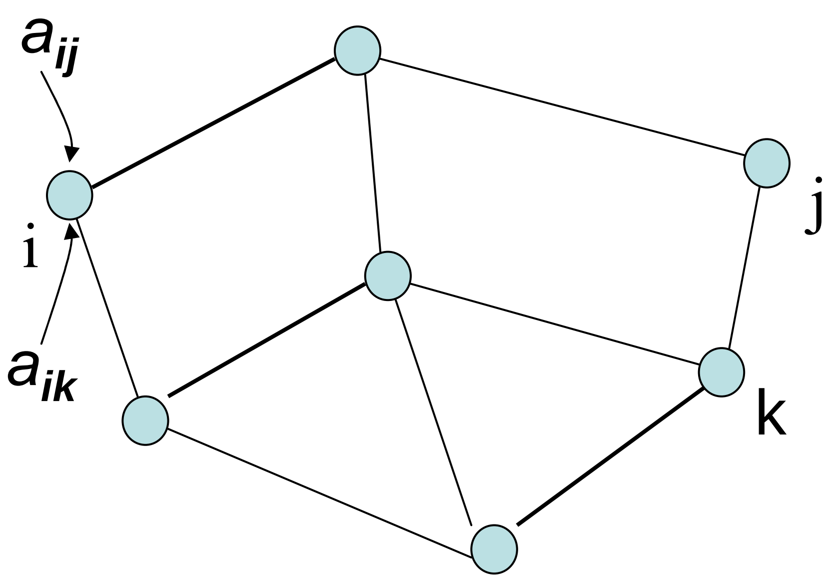
# Multihop wireless cross-layer network model




- Collection of wireless nodes moving over a terrain

- Traffic may be generated at any node i with destination any other node j (or many, multicasting), not necessarily within one hop from i

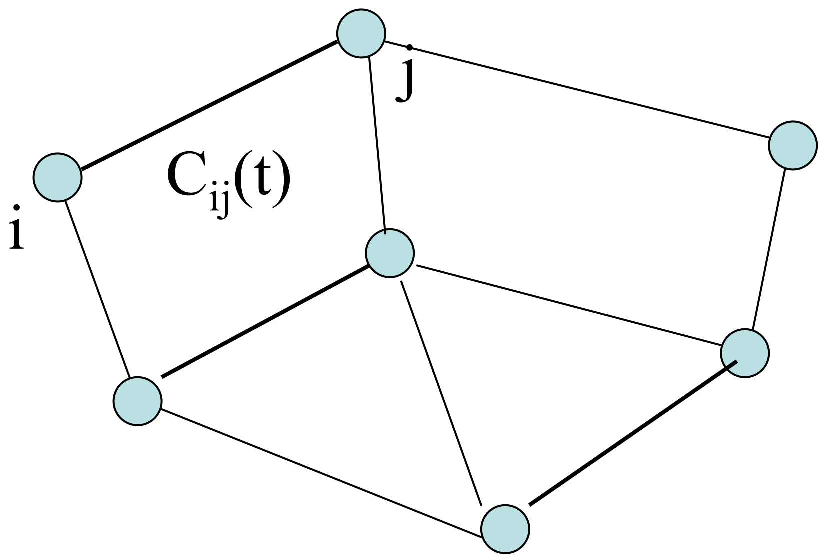
# …network model




- Nodes control transmission power, access decision (transmit, don't transmit, which code (in CDMA) etc.), other physical layer parameters represented collectively by vector I(t)

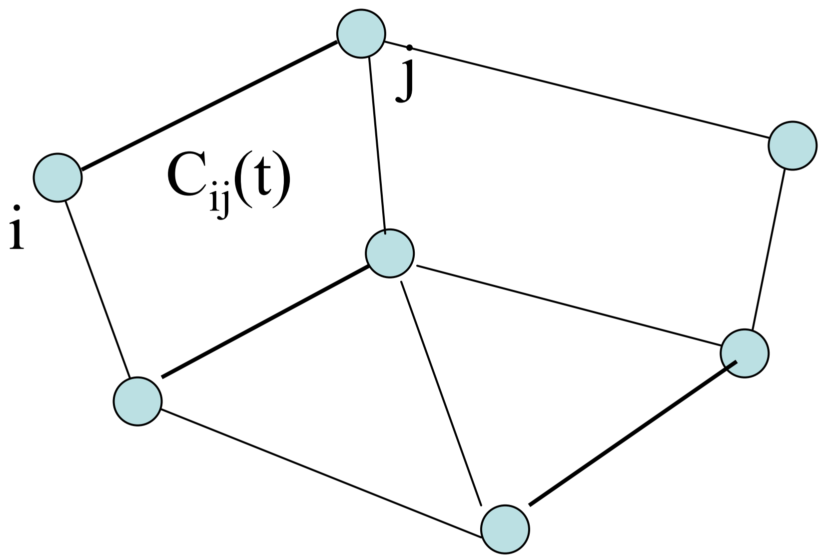
- The environment changes as well due to mobility of the nodes and the environment itself; "topology" S(t)

# …network model




- $C_{ij}$ (t)=$C_{ij}$(S(t),I(t)): rate of bit pipe from i to j at t
- C(t) communication topology at time t determined partly by environment S(t) (uncontrollable), physical and access layer decisions I(t) (controllable)

# …network model




- Multiple traffic classes 1,..,N, distinguished based
  on our objective.

- Network layer decision R(t):
  which traffic class through (i,j),
  or how to split $C_{ij}(t)$ to the different traffic classes

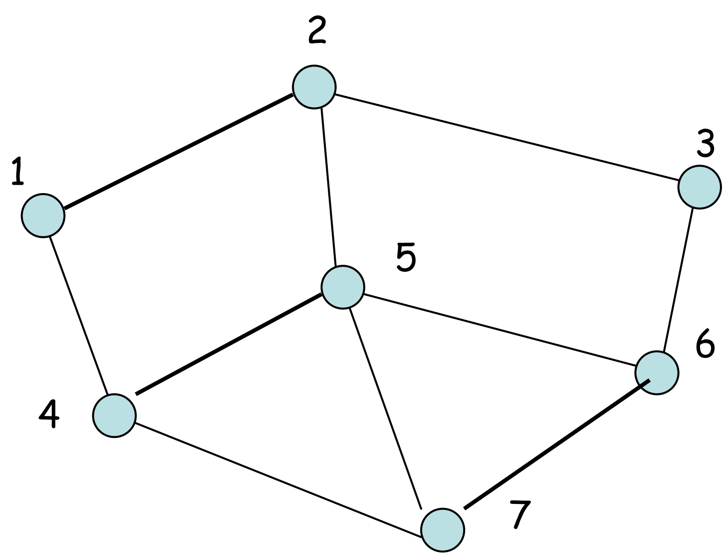
# Important Attributes-Challenges

- Radio medium, interference, need for implicit or explicit coordination at the PHY/Access layer
- Multihop traffic forwarding, routing flow control
- Both of the above functions should be accomplished under the additional complication of time-varying topology

# Interesting special cases within scope of model

- Multihop network with conflict graph based interference models
- Switch with input queueing
- Multihop network with power control and SNIR based channel model

# Conflict graph based interference models-Access layer




**Connectivity graph:** indicates pairs of nodes that can communicate directly

**Conflict graph:** indicates pairs of links constrained to communicate simultaneously

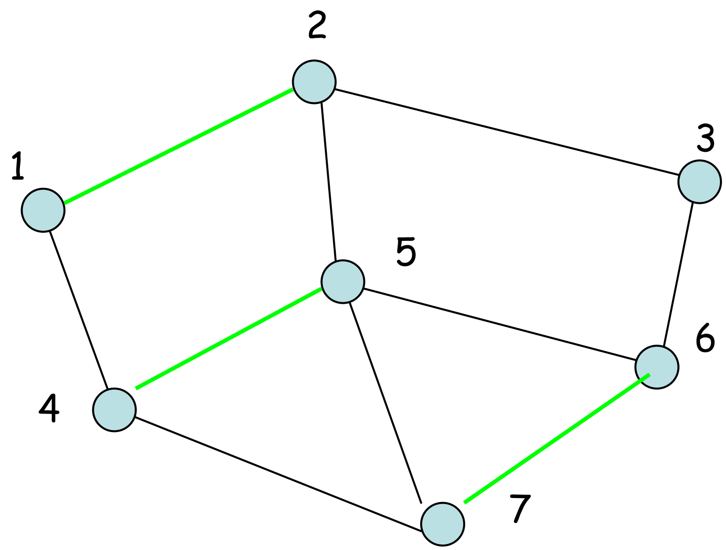
**Topology state S(t**): the connectivity graph at t

**Access Control I(t):** indicates links communicating simultaneously at t

Should be independent set of the topology graph to comply with the constraints

**C(S(t),I(t)):** indicator function of realized transmissions

# Special case: single transceiver per node constraint

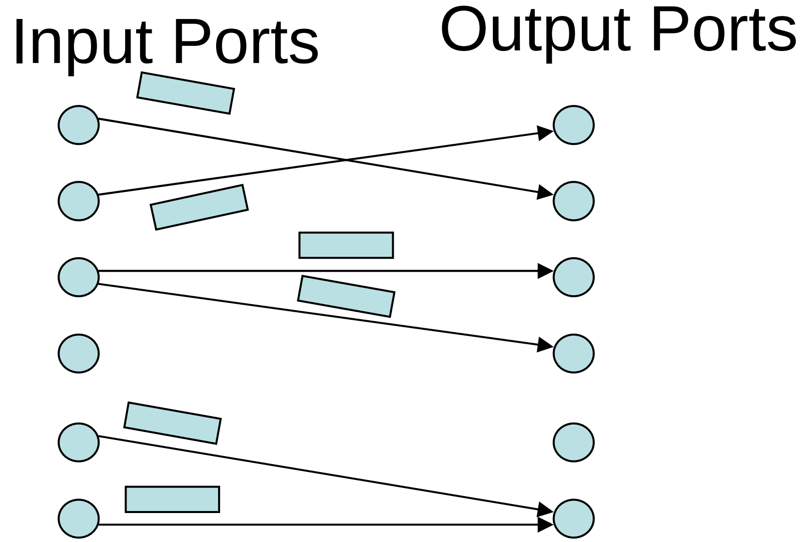



Two edges conflicting only if they share a common node

I(t) takes values in the space of matching of the connectivity graph

# Even more special case:Input Queued Switches

•Topology fixed with time, bipartite graph, single-hop traffic

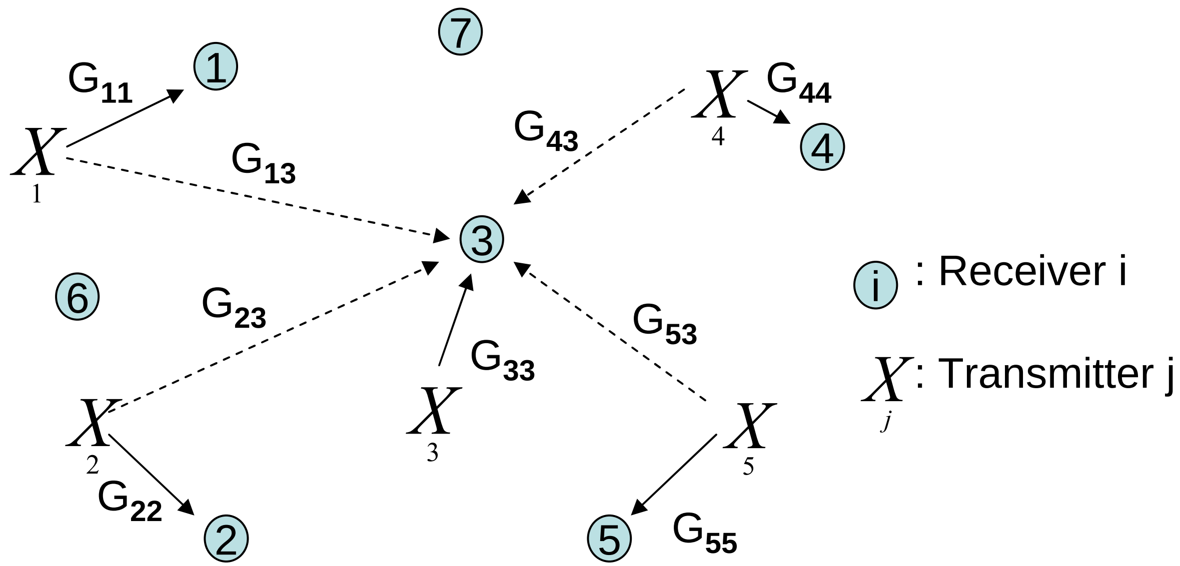



•Packets are generated at input ports, need to reach output ports

• Transmission from node *i* to node *j engages* both nodes *i* and *j*

• Parallel transmissions are allowed if they involve disjoint origin destination pairs

# Power controlled multihop network




- $G_{ji}$ (t): Path gain between transmitter *j* and receiver *i*
- $P_j$ (t): Transmitted power from transmitter *j*
- $G_{ii}$(t) $P_i$(t) : Signal power at receiver *i*
- $G_{ji}$(t) $P_j$(t) : Interfering power at receiver i from transmitter *j*
- $N_i$ : Thermal Noise at receiver *i*
- Quality metric of link *i* : $SIR_i = \dfrac{G_{ii}(t)P_i(t)}{\sum_{j\neq i} G_{ji}(t)P_j(t) + N_i} = \gamma_i$

# Power control (..continued)

I(t)=P(t), S(t)=G(t)

The rate of link i is

$$C_i(t) = C(G(t), P(t)) = C\left(\frac{G_{ii}(t)P_i(t)}{\sum_{j\neq i} G_{ji}(t)P_j(t) + N_i}\right) =$$

For AWGN channels $C(G(t), P(t)) = \log\left(1 + \frac{G_{ii}(t)P_i(t)}{\sum_{j\neq i} G_{ji}(t)P_j(t) + N_i}\right)$

MIMO systems: a similar formula holds

$$C_i(t) = C(G(t), P(t), W(t))$$

where W(t) is the beamforming weight vector

# Traffic flow considerations

$a_i^j(t)$: amount of traffic generated at node i for j in the interval [0,t] (arrivals)

$a_{ik}^j(t)$: amount of traffic destined to j, transmitted from node i to node k in the interval [0,t] (cross traffic)

$x_i^j(t)$: traffic destined to node j, accumulated in i at t

Flow conservation of traffic class j at node i, at t

$$\sum_{k=1}^{N} a_{ki}^j(t) = x_i^j(0) + x_i^j(t) + \sum_{k=1}^{N} a_{ik}^j(t)$$

# Radio link capacity condition

$\mathrm{a}_{\mathrm{ik}}^{\mathrm{j}}(t_2) - \mathrm{a}_{\mathrm{ik}}^{\mathrm{j}}(t_1)$ : Amount of class j traffic crossed link (i,j) in time interval $(t_1, t_2)$

$$\sum_{\mathrm{j}} (\mathrm{a}_{\mathrm{ik}}^{\mathrm{j}}(t_2) - \mathrm{a}_{\mathrm{ik}}^{\mathrm{j}}(t_1)) \le \int_{t_1}^{t_2} C_{ik}(S(t), I(t))dt$$

Network control policy {(I(t),R(t)): t=1,2,..}

# Stability

- Stochastic traffic

$$\sup_{\{t>0\}} E[X_{ij}(t)] < \infty$$

- Deterministic traffic

$$\sup_{\{t>0\}} X_{ij}(t) < \infty$$

## Necessary condition for stability

Assuming arrivals, cross traffic and capacity have long term avg.

$$\lim_{t\to\infty} a_i^j(t)/t = a_{ij}, \quad \lim_{t\to\infty} a_{ik}^j(t)/t = f_{ik}^j,$$

$$\lim_{t\to\infty} \frac{1}{t}\int_0^t C_{ik}(S(t), I(t))dt = C_{ik} \quad \text{a.s.}$$

**Flow conservation** at each node i for each traffic class m

$$\sum_{k=1}^{N} f_{ki}^m + a_{im} = \sum_{j=1}^{N} f_{ij}^m$$

(if not then class m backlog of node i will grow to infinity)

**Link capacity** condition

$$\sum_{m=1}^{N} f_{ij}^m < C_{ij}$$

# Feasible link rate topologies at the access layer

Rate vector for some fixed state S(t)=s and access policy I(t)

$$C = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} C(s, I(t)) , \quad I(t) \in K$$

Capacity region C(s) for fixed topology state s includes all rate vectors realized by any access policy

C(s) the convex hall of {C(s,I): I in K}

Capacity region C the expectation of C(s) with respect to the stationary distribution of topology process S(t) i.e.

**C**={C: C=E[C(s)], C(s)ε **C**(s)}

# Throughput Consideration

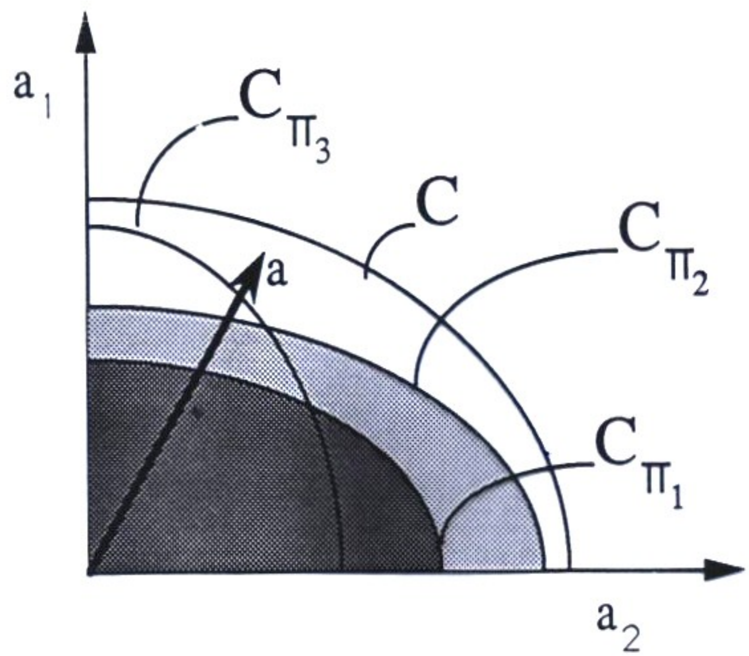
•Traffic load vector A includes all origin-destination pairs arrival rates

•Capacity region Cπ of a policy π: the set of traffic load vectors A for which the system is stable under π

•Capacity Region C of the system:

$$C = \bigcup_{\pi} C_{\pi}$$

π network control policy




Design objective:
*Obtain policies with large capacity regions, for robust operation to unpredictable variations of the traffic and the environment*

# Backpressure mechanism for routing and flow control

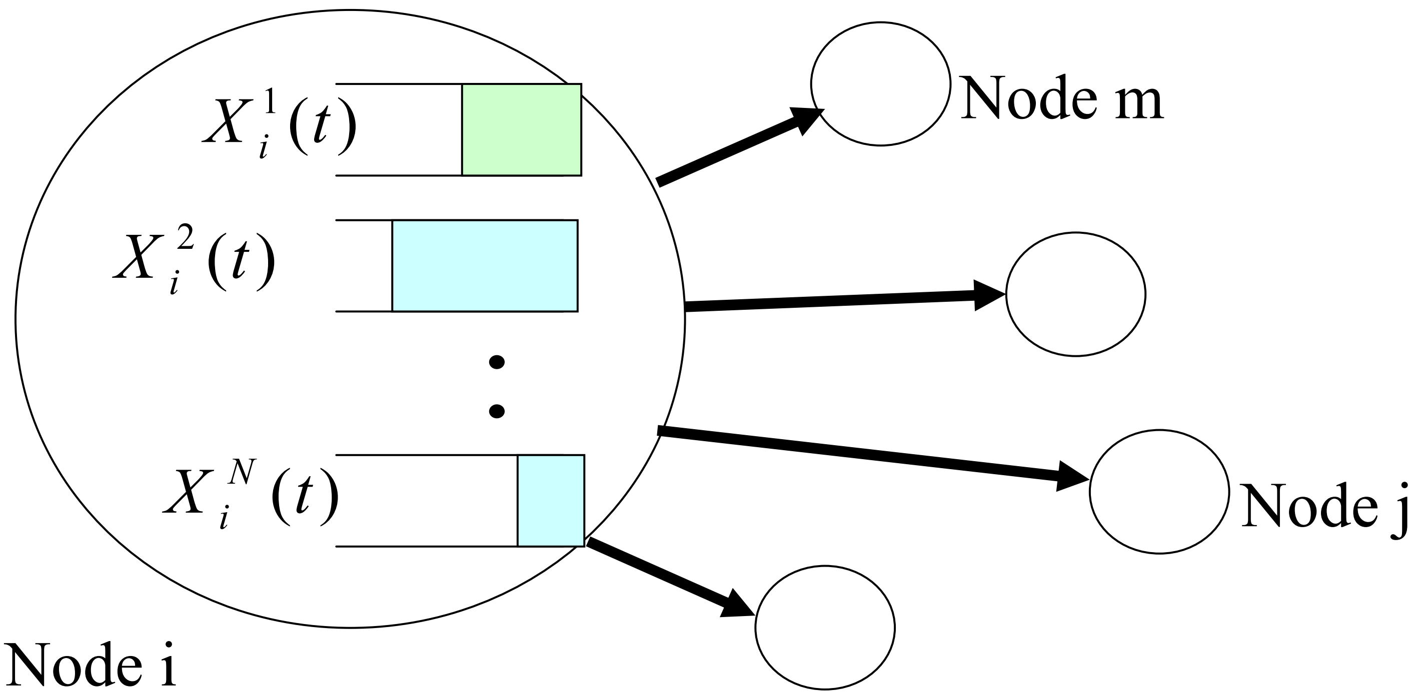
*Datagram traffic forwarding*

A packet in transit is characterized by its destination alone

At each node packets of N traffic classes, one for each destination

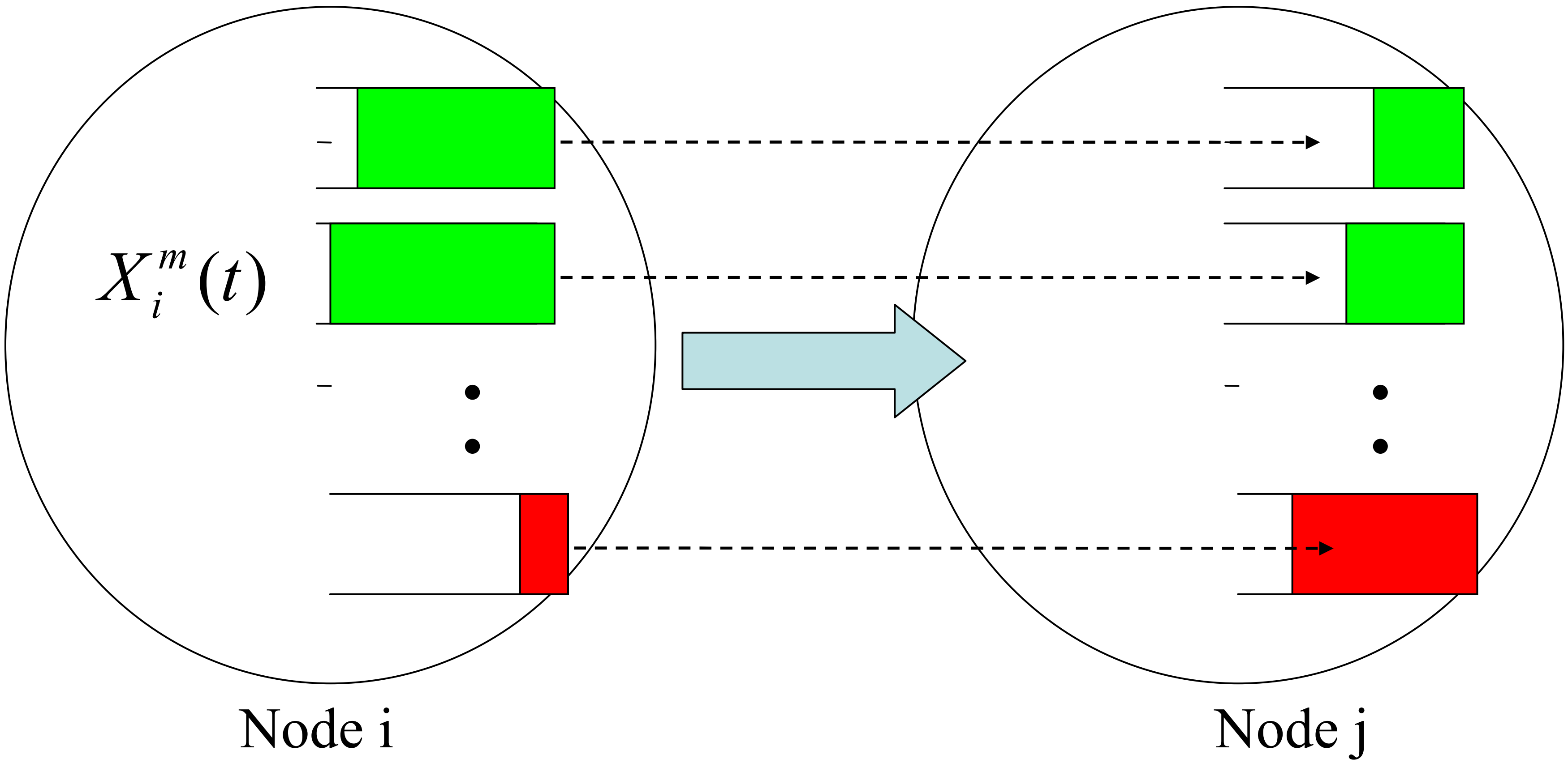
One packet may be forwarded through each link

$R_{ij}(t)$ : class of packet through link (i,j) at t, or 0 if no transmission



# Back pressure flow control

If $X_i^m(t) - X_j^m(t)$ is negative then class m is no eligible for transmission from i to j

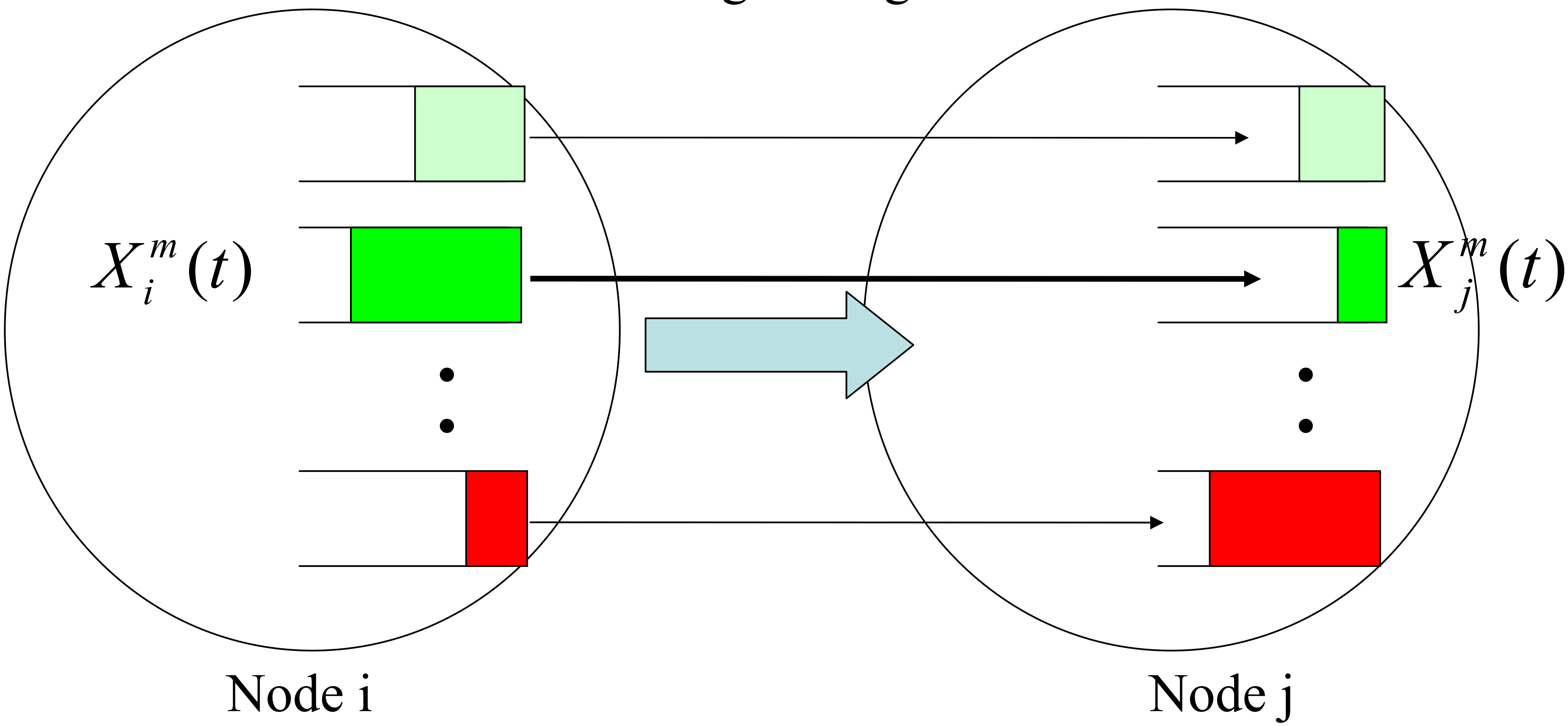


## Class priority scheduling

Transmit a packet of class m for which

$$X_i^m(t) - X_j^m(t)$$

is maximum among all eligible classes



**The combination of backpressure flow control with class priority scheduling achieves maximum traffic forwarding throughput in the datagram network**

**Furthermore it is inherently distributed and computationally simple**

# Maxweight scheduling at the MAC/PHY layer for maximum throughput.

- Consider a single hop network like the switch for instance
- **Max weight access control** policy selects I(t) to maximize

$$X(t)*C(S(t),I(t))$$

X(t) vector of packet backlog for each link

maxweight achieves maximum throughput

# Access control jointly with traffic forwarding

Select I(t) to maximize the following objective

$$\sum_{i,j=1}^{N} w_{ij} C_{ij}(S(t), I(t))$$

where

$$w_{ij} = \max_{m=1..N} \left\{ X_m^i(t) - X_m^j(t) \right\}$$

**The joint scheme above achieves max end-to-end throughput**

# Dealing with complex optimization problems

Crucial step: select I(t) to maximize

$$\sum_{i,j=1}^{N} w_{ij} C_{ij}(S(t), I(t))$$

- The optimization problem depends on the physical application, its complexity may vary from sublinear to NP-hard
- The parameters of the optimization might be distributed to different nodes physically separated and the problem inherently distributed
- Some of the parameters-state of the system might be partially or non-observable
- In several occasions the computational resources of the system might be severely limited (sensor networks)

# Randomization against the complexity curse

- A randomized algorithm for selecting I(t) is represented by a probability distribution *P(X,.)* on *K*, parameterized by the weights *X*

- Consider randomized algorithms with the property: if *i* has distribution *P(X,.),* the

$$(C): \ P(X, \underset{I}{\operatorname{argmax}}(X\, I^T)) \geq \in > 0, \ \forall X$$

- Simple randomized algorithm with the above property: Select each $I_{ij}$ by flipping a fair coin. If the resultant vector is a matching, then this is *I*. Otherwise *I = 0*.

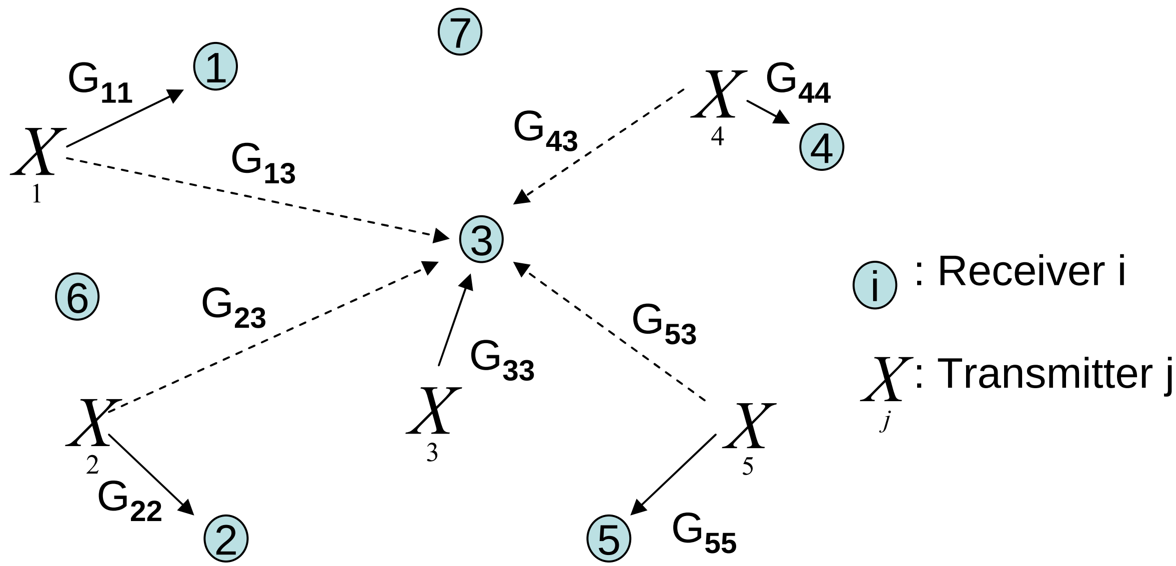
  - Property *C* holds with $\in = (½)^{NM}$

## Randomized Scheduling

- The following randomized scheduling policy with memory achieves maximum throughput
- Let $I(t)$, $t = 1,2,\ldots$ be a sequence of independent random vectors with distribution $P(X(t),.)$ at $t$

$$I(t) = \begin{cases} \hat{I}(t) & \text{if } X(t)^T \hat{I}(t) > X(t)^T I(t-1) \\ I(t-1) & \text{Otherwise} \end{cases}$$

# Power controlled multihop network




- $G_{ji}$ (t): Path gain between transmitter *j* and receiver *i*
- $P_j$ (t): Transmitted power from transmitter *j*
- $G_{ii}$ (t) $P_i$ (t) : Signal power at receiver *i*
- $G_{ji}$ (t) $P_j$ (t) : Interfering power at receiver i from transmitter *j*
- $N_i$ : Thermal Noise at receiver *i*
- Quality metric of link *i* : $SIR_i = \dfrac{G_{ii}(t)P(t)_i}{\sum_{j \neq i} G_{ji}(t)P_j(t) + N_i} = \gamma_i$

# Maximum throughput power control policy

$$\max_{P(t)} \sum_i X_i(t) \log\left(\frac{G_{ii}(t)P_i(t)}{\sum_{j\neq i} G_{ji}(t)P_j(t) + N_i}\right)$$

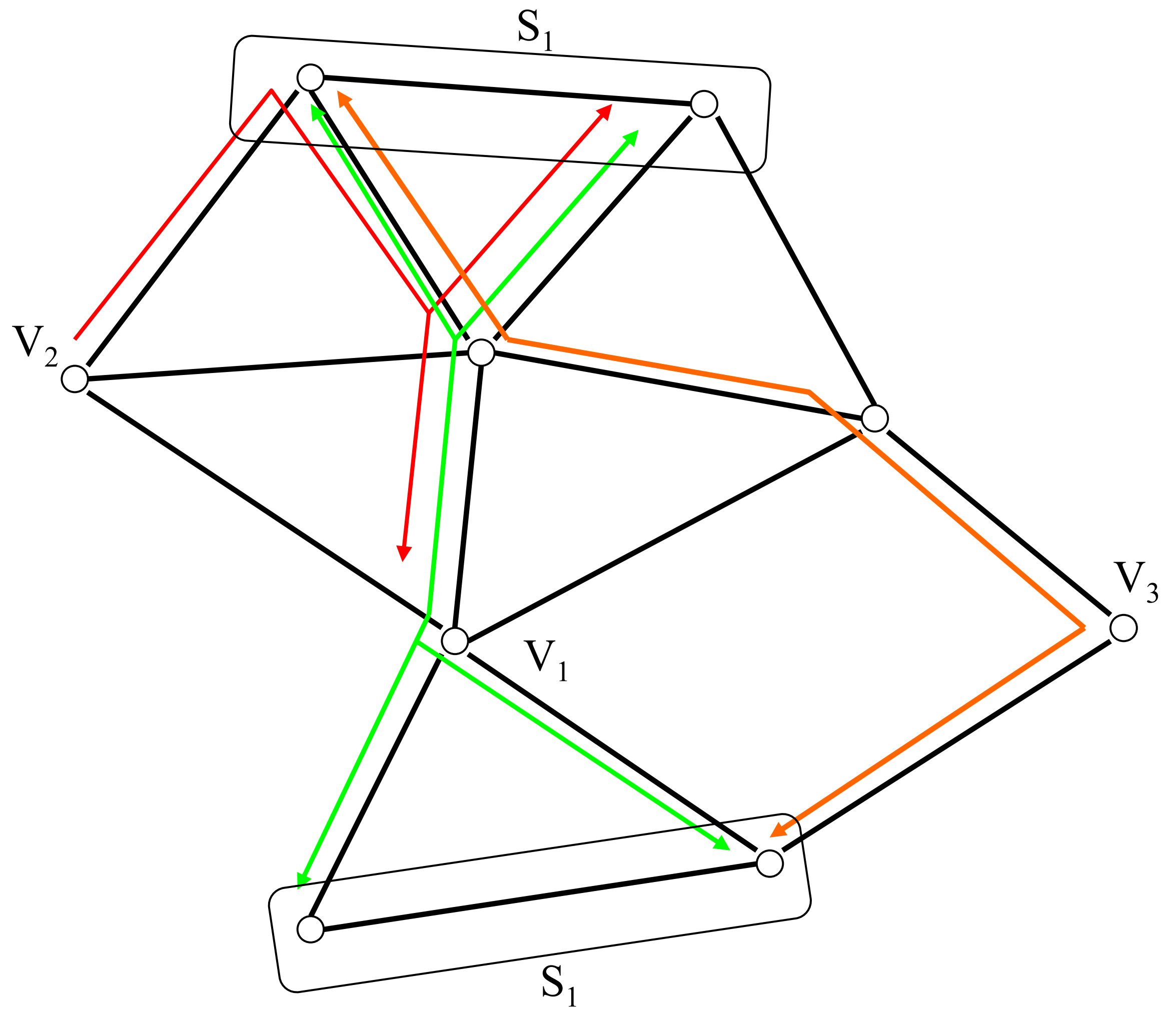
The optimization problem is solvable by gradient projection type of algorithms in certain cases, that might be amenable to distributed implementations in certain cases.

We have shown recently that performing even one iteration per slot of the optimization algorithm is adequate to achieve maximum throughput.

Opens a direction for implementable algorithms for maximum throughput

# Multicasting

- Consider $N$ multicast sessions $(v_1,S_1)$, $(v_2,S_2)$,…,$(v_N,S_N)$
  $v_\mathrm{n}$ : Information Source
  $S_\mathrm{n}$ : Group of intended destinations for information source $v_\mathrm{n}$
- $\tau_n$ : Collection of directed trees rooted at $v_n$ with leaves ending in the set of nodes $S_n$ that may carry session $n$ traffic
- $\tau_n$ may include
  - All multicast trees routed at $v_n$ with leaves terminating in $S_n$
  - Some pre-selected multicast trees.
- $a_\mathrm{n}$ : traffic rate of session n, split among the trees of $\tau_n$




**One multicast tree per session is depicted, there are three sessions**

## Necessary and sufficient throughput feasibility condition

A collection of traffic rates $a_n,\ n = 1,2,...,N$ is feasible if there exist a traffic splitting $a_n^m, m = 1,2,..., M_n$ for each session n,

$$a_n = \sum_{m=1}^{M_n} a_n^m T_n^m$$

such that the capacity condition is satisfied

$$\sum_{n=1}^{N} \sum_{m=1}^{M_n} (a_n^m T_n^m) \leq C$$

$$C = (C_e : e \in E)$$

$$C_e : \text{Capacity of link } e$$

$T_n^m$ : The $m^{th}$ multicast tree that may carry session $n$ traffic represented by a binary indicator vector $T_n^m = (t_e : e \in E)$

**Verifying feasibility NP-hard, Steiner tree packing problem**

# Backpressure and per link priority scheduling

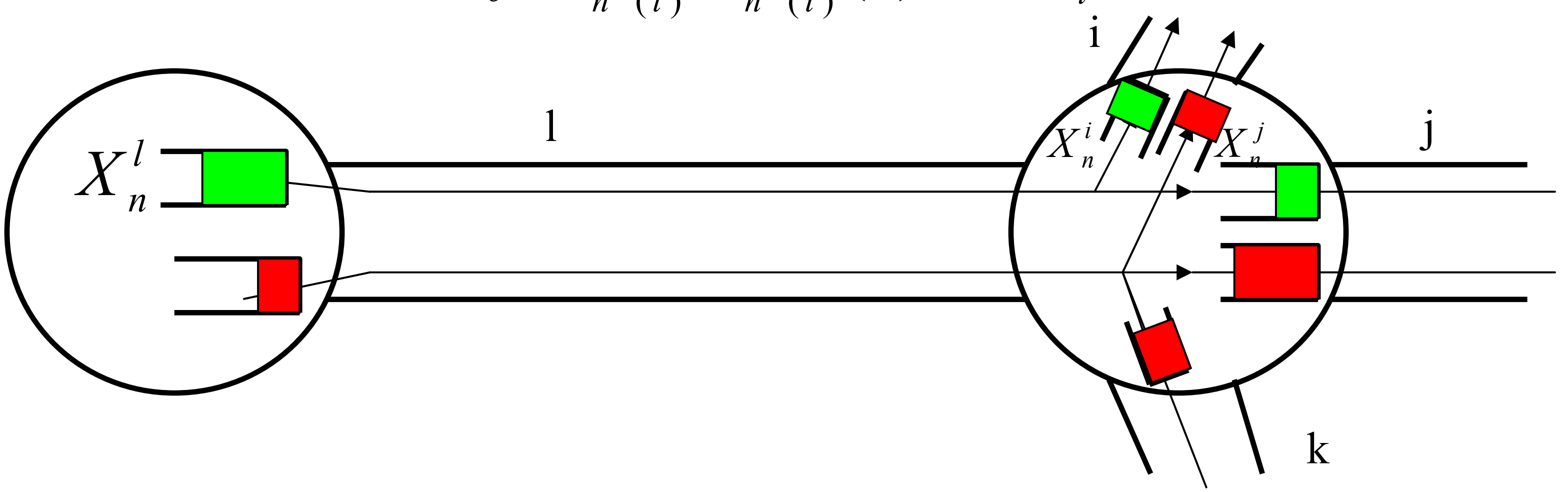
- $X_n^l(t)$ : Backlog of tree $n$ traffic in front of link l

$$W_n^l(t) = X_n^l(t) - \max_{\text{k is a descendent of l}} X_n^k(t)$$

- $W_n^l(t)$ :Weight (backlog gradient) of tree $n$ at link $l$
- $b_n^l W_n^l(t)$ : Priority index of tree $n$ trough link $l$.

$$n^l(t) = \arg\max_n b_n^l W_n^l(t)$$

$$if \;\; b^l_{n^l(t)} W^l_{n^l(t)}(t) < -T_l \;\; then \;\; idle$$



## Traffic splitting among trees at the source: Load balancing

**Rule 1:** at the source node the traffic is assigned to the multicast tree with minimum local backlog

**Rule 2:** at the source node the traffic is assigned to the multicast tree with minimum weight, where the weight of a tree is the sum of the weights of its links and the weight of a link is the maximum traffic backlog through the link.

The combination of the link scheduling prioritization scheme with either of the load balancing rules for traffic assignment achieve maximum throughput

# Recent and ongoing research

- Approximation algorithms for the maxweight problem in the conflict graph interference model with provable throughput performance
- "Distributization" of the algorithms
- Exploration of the randomization approach
- Consider a utility maximization approach combining backpressure with rate control at the edge in order to tackle objectives beyond throughput, i.e fairness, delay, energy consumption, etc.
- Combine back-pressure based control with network coding

Represents work of several people, including non-exhaustively: Stolyar, Neely, Modiano, Shroff, Lin, Srikant,Eryilmaz, Sarkar, Yi, Chiang, Proutiere, Shah, Yeh, Prabhakar, Neri, Giaconne