Coded Caching for Content Distribution

Urs Niesen

MobiHoc 2018

Importance of Content Distribution

- Video on demand is driving network traffic growth
 - Netflix streaming service, Amazon Prime Video, Hulu, Verizon
 / Comcast on Demand, . . .
- IP video traffic is predicted to make up 82% of all IP traffic by 2021¹

¹Cisco, "The Zettabyte era: Trends and analysis," Tech. Rep., Jun. 2017.

Importance of Content Distribution

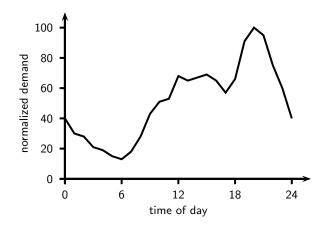
- Video on demand is driving network traffic growth
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- Places significant stress on service provider's networks

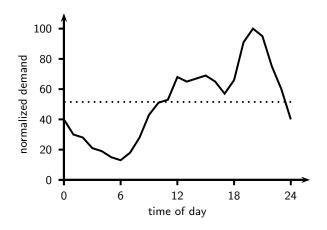
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Importance of Content Distribution

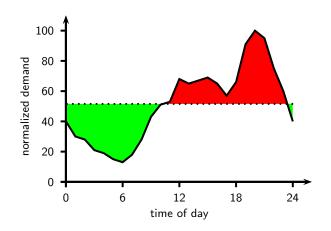
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- IP video traffic is predicted to make up 82% of all IP traffic by 2021¹
- Places significant stress on service provider's networks
- Caching (prefetching) can be used to mitigate this stress

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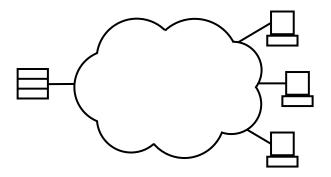


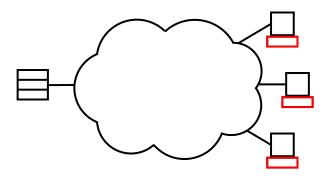


■ High temporal traffic variability

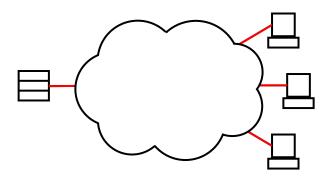


- High temporal traffic variability
- Caching can help smooth traffic





■ Placement phase (5am): Populate caches



- Placement phase (5am): Populate caches
- Delivery phase (8pm): Request and deliver movies

Conventional beliefs about caching:

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This talk will argue:

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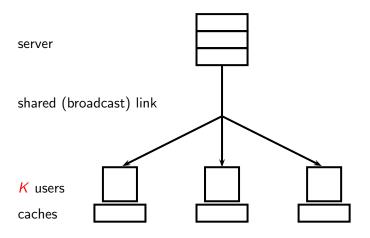
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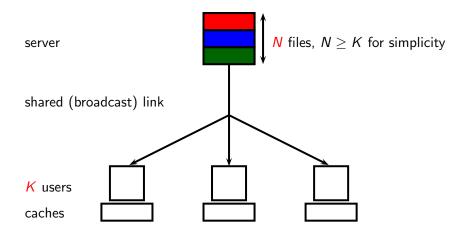
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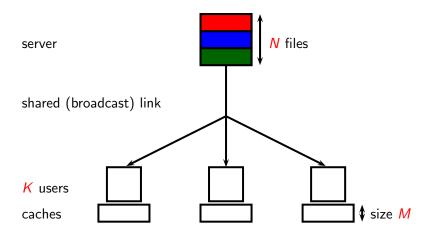
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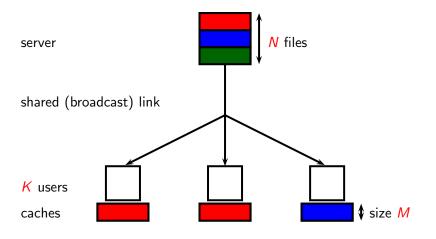
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- The main gain in caching is global
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- Statistically identical users ⇒ different cache content
- Coded multicasting as key enabler

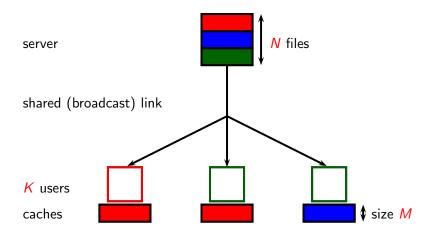




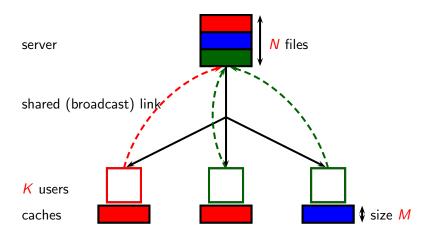




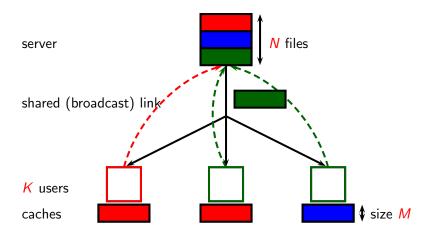
Placement: cache arbitrary function of files (linear, nonlinear, ...)



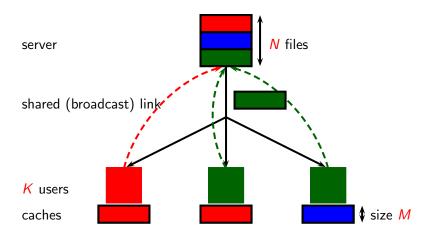
Delivery:



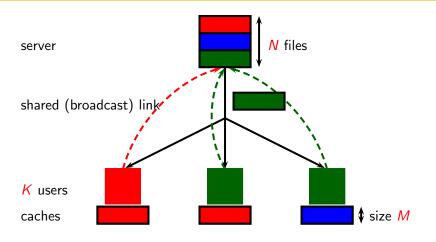
Delivery: - requests are revealed to server



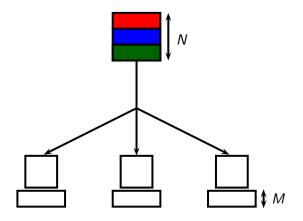
Delivery: - requests are revealed to server - server sends arbitrary function of files

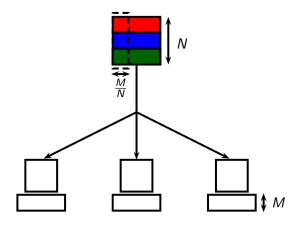


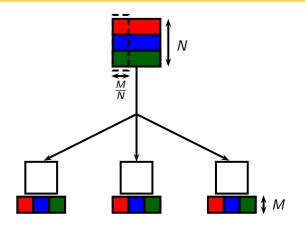
Delivery: - requests are revealed to server - server sends arbitrary function of files

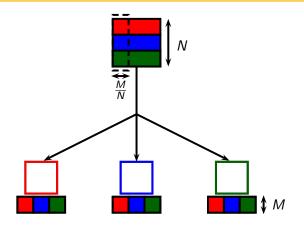


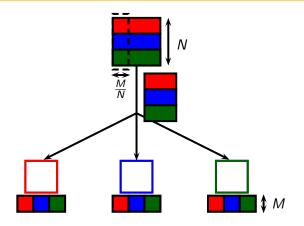
Question: smallest worst-case rate R(M) needed in delivery phase?

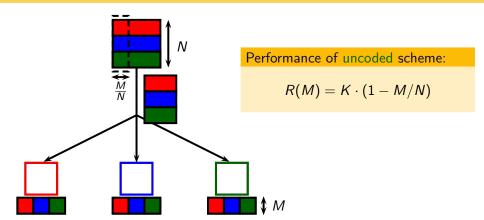


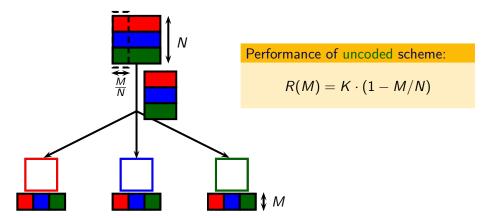




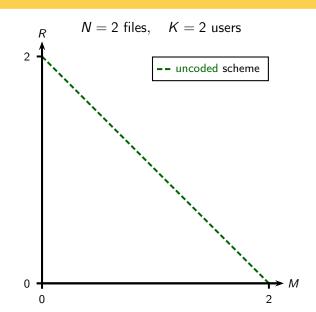


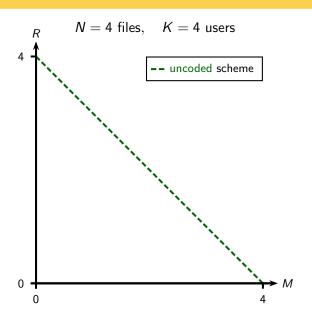


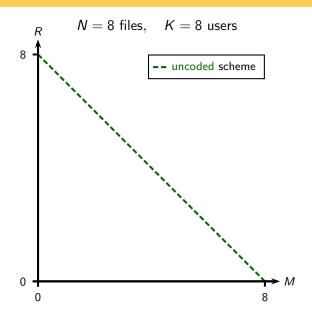


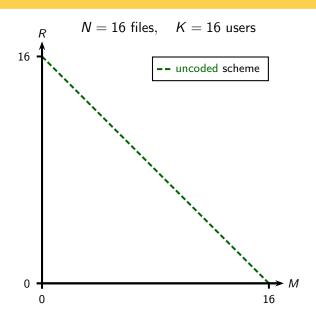


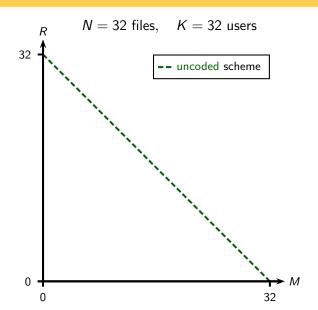
- lacktriangle Caches provide content locally \Rightarrow local cache size matters
- Identical cache content at users

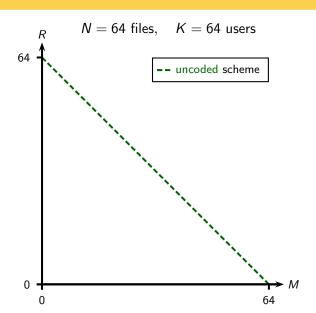


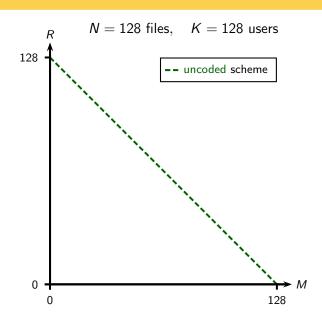


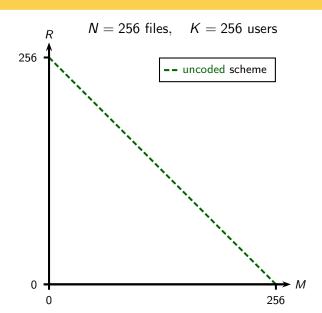


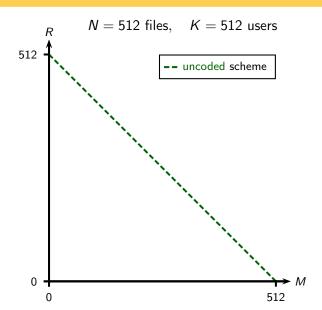












Proposed Coded Caching Scheme N files, K users, cache size M

- Design guidelines advocated in this talk:

 The main gain in caching is global
 - Global cache size matters
 - Different cache content at users
 - Coded multicasting

N files, K users, cache size M

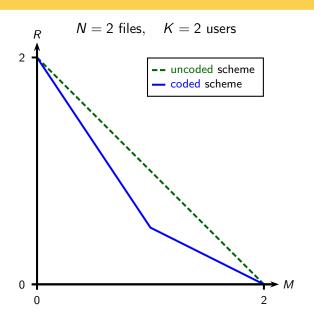
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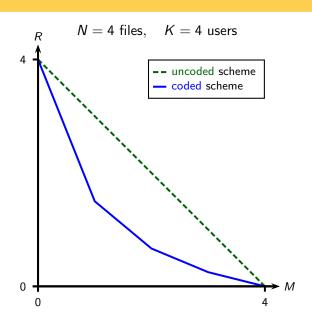
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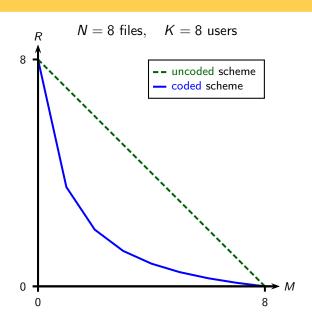
Performance of coded scheme:²

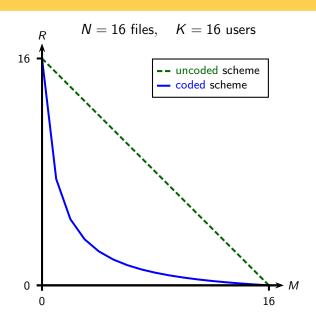
$$R(M) = K \cdot (1 - M/N) \cdot \frac{1}{1 + KM/N}$$

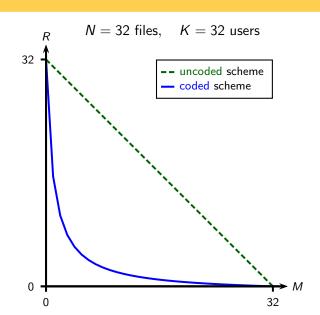
²M. A. Maddah-Ali and U. Niesen, "Fundamental limits of caching," *IEEE Trans. Inf. Theory*, vol. 60, no. 5, pp. 2856–2867, May 2014.

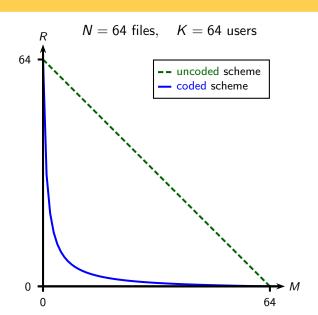


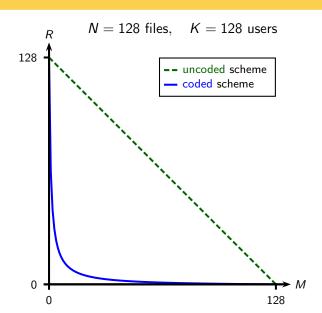


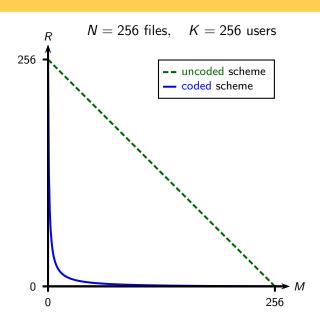


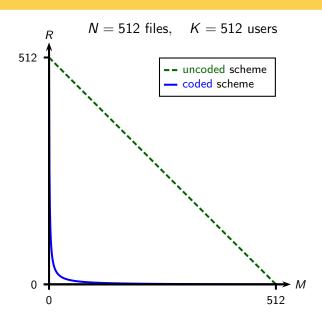


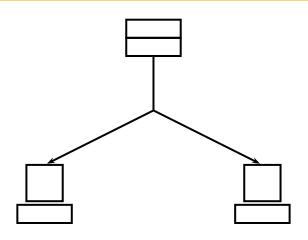


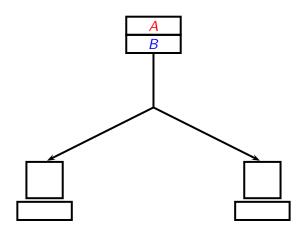


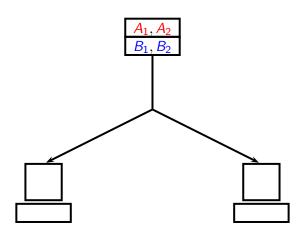


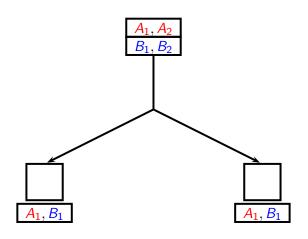


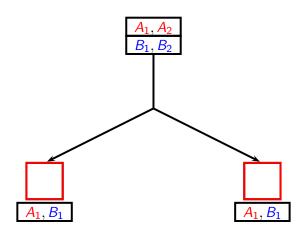


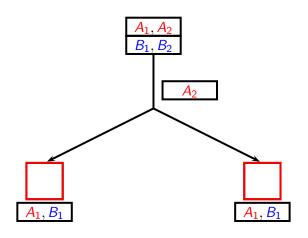


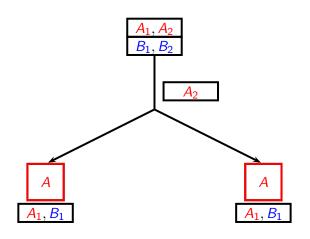


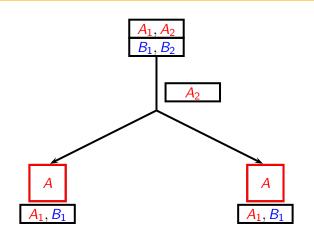




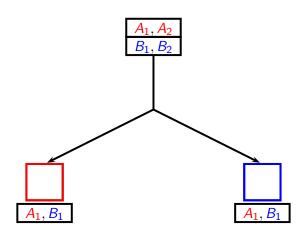


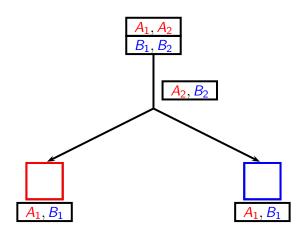


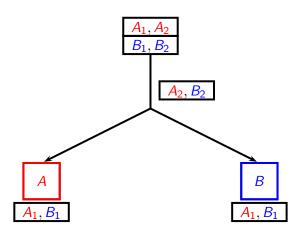




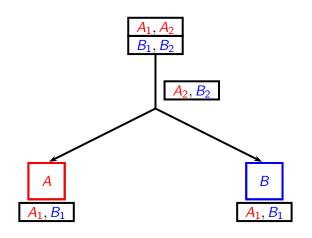
- ⇒ Identical cache content at users
- ⇒ Gain from delivering content locally





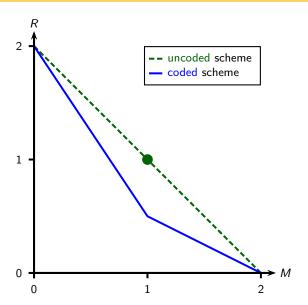


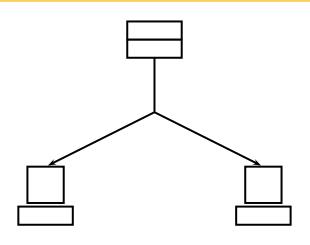
N=2 files, K=2 users, cache size M=1

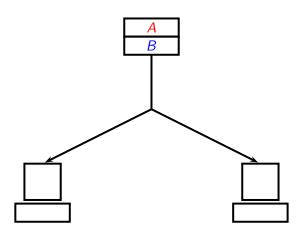


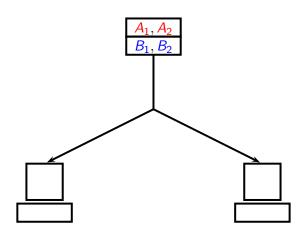
 \Rightarrow Multicast only possible for users with same demand

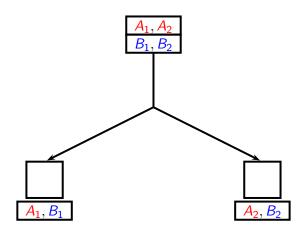
Recall: Uncoded Scheme N = 2 files, K = 2 users, cache size M = 1

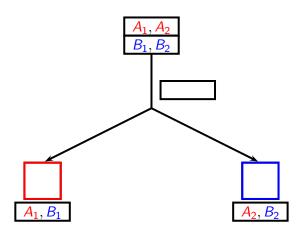


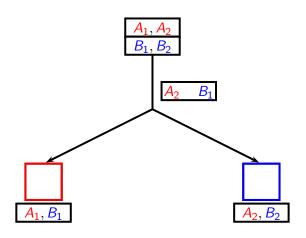


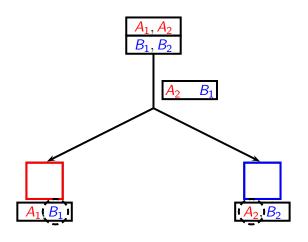


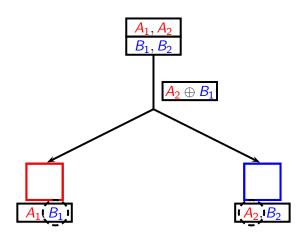


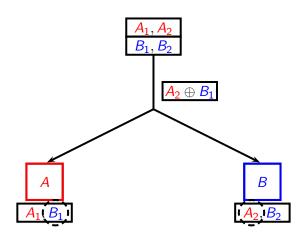


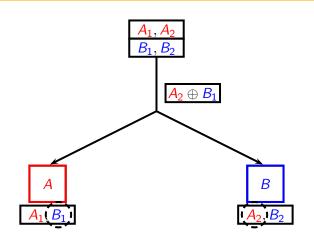




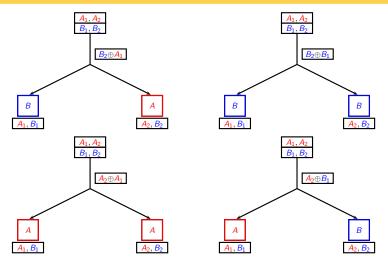






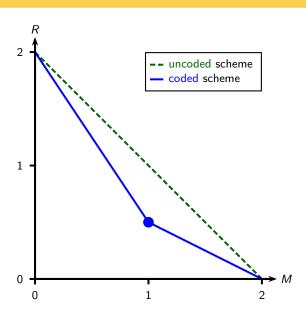


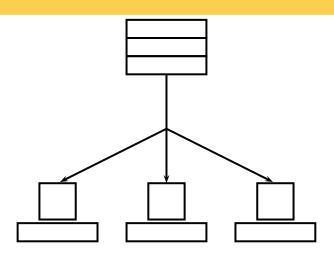
- ⇒ Different cache content at users
- ⇒ Coded multicast to 2 users with different demands

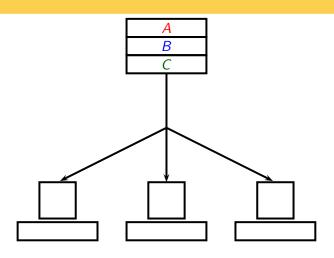


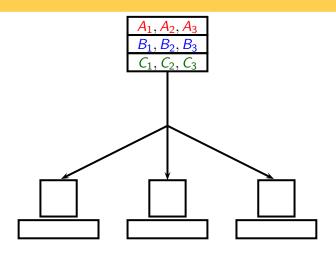
- ⇒ Works for all possible user requests
- ⇒ Simultaneous coded multicasting gain

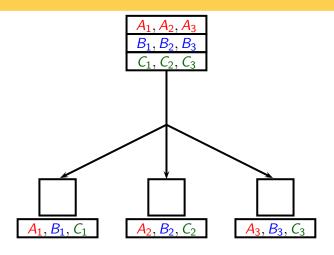
Proposed Coded Scheme N = 2 files, K = 2 users, cache size M = 1

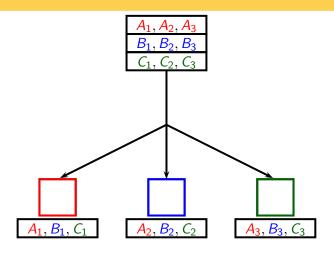


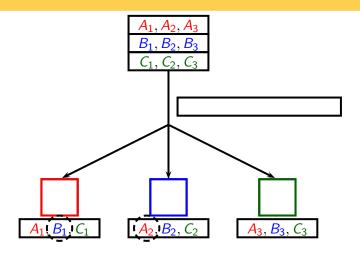


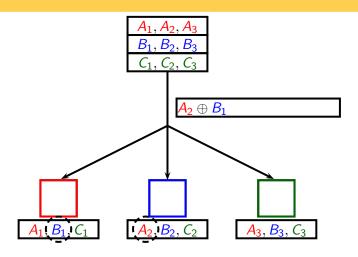


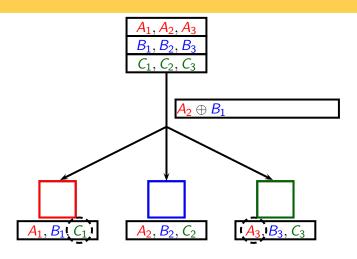


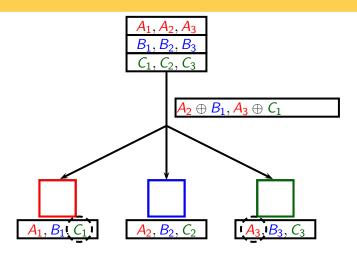


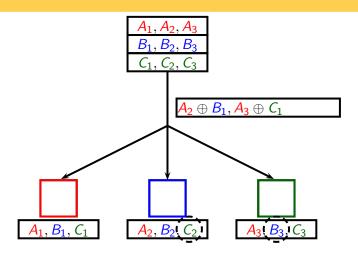


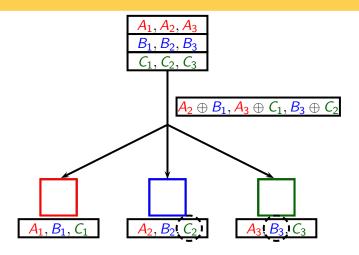


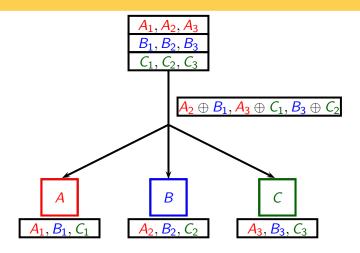




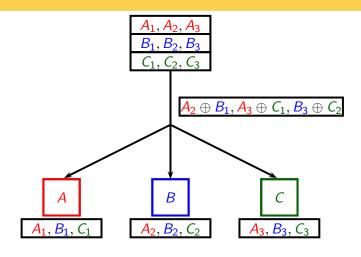




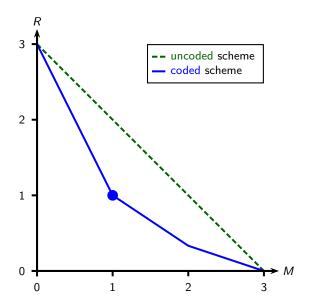


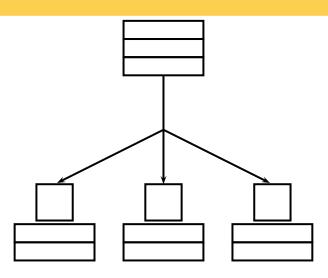


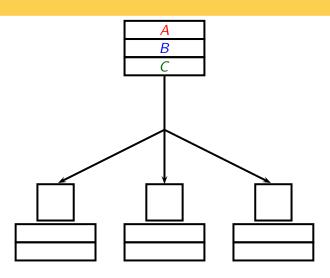
N=3 files, K=3 users, cache size M=1

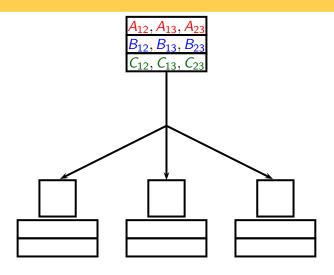


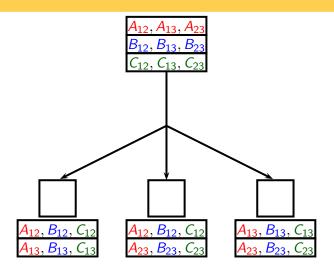
⇒ Coded multicast to 2 users with different demands

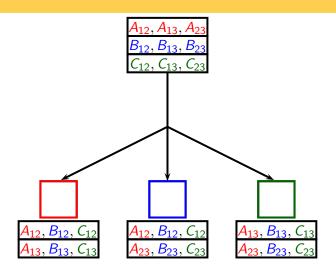


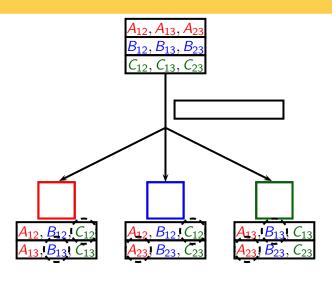


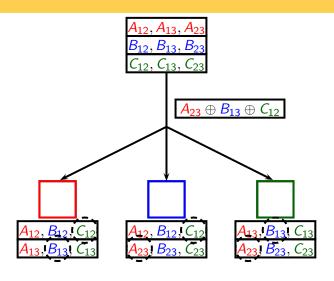


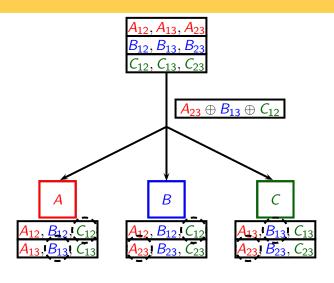




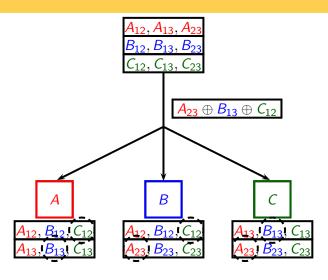






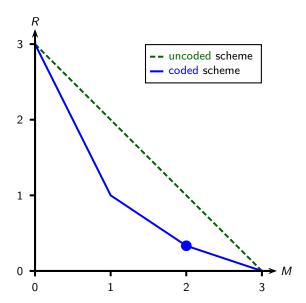


N=3 files, K=3 users, cache size M=2



⇒ Coded multicast to 3 users with different demands

Proposed Coded Scheme N = 3 files, K = 3 users, cache size M = 2

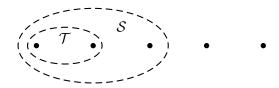


Proposed Coded Scheme N = K files and users, cache size M

■ Goal: coded multicast to M + 1 users with different demands

N = K files and users, cache size M

- Goal: coded multicast to M + 1 users with different demands
- Need to place content such that in delivery phase:
 - 1 for every possible user demands...
 - **2** and for every possible subset S of M+1 users...
 - 3 and for every possible subset $\mathcal{T} \subset \mathcal{S}$ of M users. . .
 - 4 users in ${\mathcal T}$ share content that is required at the user in ${\mathcal S} \setminus {\mathcal T}$



N = K files and users, cache size M

- Goal: coded multicast to M + 1 users with different demands
- Need to place content such that in delivery phase:
 - 1 for every possible user demands...
 - 2 and for every possible subset S of M+1 users. . .
 - 3 and for every possible subset $\mathcal{T} \subset \mathcal{S}$ of M users. . .
 - 4 users in ${\mathcal T}$ share content that is required at the user in ${\mathcal S} \setminus {\mathcal T}$

Example: N = K = 3, M = 2

Every two users have a piece of content the remaining user needs

Proposed Coded Scheme N = K files and users, cache size M

Placement phase:

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■ N files: $W_1, ..., W_N$

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- Cache k: $(W_{n,\mathcal{T}}: n \in [N], \mathcal{T} \subset [K], |\mathcal{T}| = M, k \in \mathcal{T})$

Example:
$$N = K = 3$$
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Consider files $A, B, C \Rightarrow \text{ cache 2: } (A_{12}, A_{23}, B_{12}, B_{23}, C_{12}, C_{23})$

Delivery phase:

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Example:
$$N = K = 3$$
, $M = 1$

Consider files A, B, C and user requests $d_1 = A, d_2 = B, d_3 = C$

 \Rightarrow Server sends $A_2 \oplus B_1$, $A_3 \oplus C_1$, $B_3 \oplus C_2$

Comparison of the Two Schemes N files, K users, cache size M

- Uncoded scheme: $R(M) = K \cdot (1 M/N)$
- Coded scheme: $R(M) = K \cdot (1 M/N) \cdot \frac{1}{1 + KM/N}$

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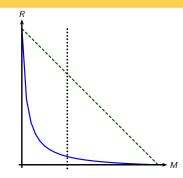
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- \Rightarrow Global gain can be $\Theta(K)$ smaller than local gain

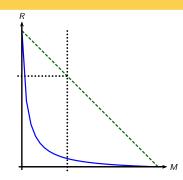
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$$N = 30$$
 files, $K = 30$ users, cache size $M = 10$

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$$R(M) = K \cdot (1 - M/N)$$
$$\approx 30 \cdot 0.67 \approx 20$$



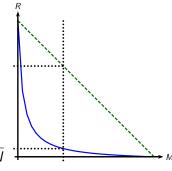
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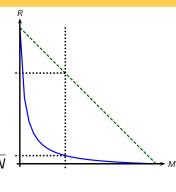
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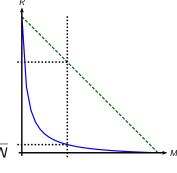
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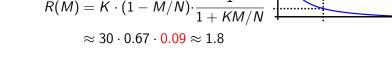
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- ⇒ Factor 11 reduction in rate!
- \Rightarrow Local gain is 0.67
- \Rightarrow Global gain is 0.09 (coded multicast to M+1=11 users with different demands)

Theorem

The coded scheme is optimal to within a constant factor in rate.³

³M. A. Maddah-Ali and U. Niesen, "Fundamental limits of caching," *IEEE Trans. Inf. Theory*, vol. 60, no. 5, pp. 2856–2867, May 2014.

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- ⇒ Information-theoretic bound
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- ⇒ No other significant gain besides local and global

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Approach Can be Adapted to Handle...

- Asynchronous user requests⁴
- Nonuniform file popularities⁵
- Users joining and leaving the network⁶
- Several users sharing a cache⁷
- Online cache updates⁸
- More complicated network topologies⁹

⁴Niesen and Maddah-Ali 2015.

 $^{^5}$ Niesen and Maddah-Ali 2017; Ji, Tulino, Llorca, and Caire 2017; Zhang, Lin, and Wang 2018.

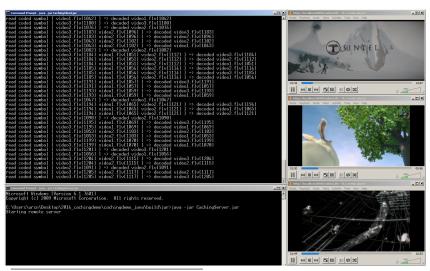
⁶Maddah-Ali and Niesen 2015.

⁷Hachem, Karamchandani, and Diggavi 2017.

⁸Pedarsani, Maddah-Ali, and Niesen 2016.

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Video Streaming Demo¹⁰



¹⁰U. Niesen and M. A. Maddah-Ali, "Coded caching for delay-sensitive content." in *Proc. IEEE ICC*, Jun. 2015, pp. 5559–5564.

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- Large scale implementation:
 - So far only demo-sized implementation
 - Experimentation with large-scale systems are needed

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Conclusions

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- Key open questions: block length, state, large-scale implementation

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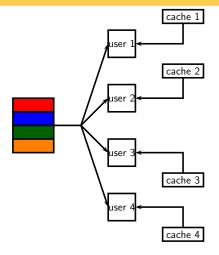
References III

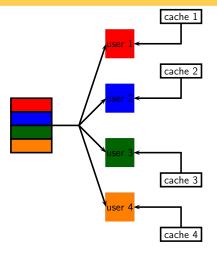


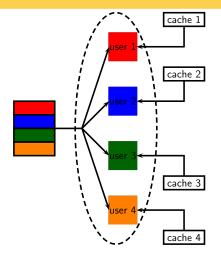
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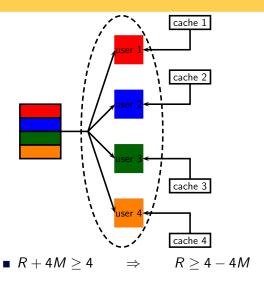


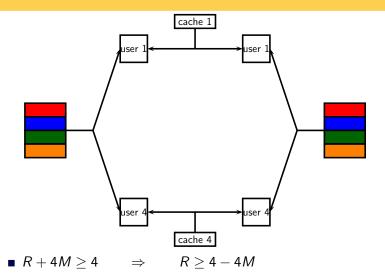
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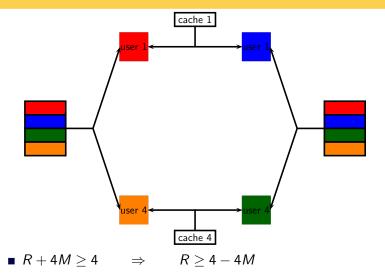


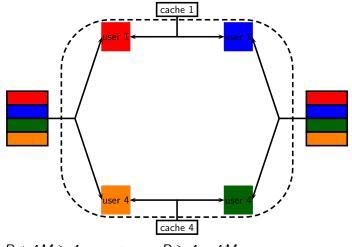






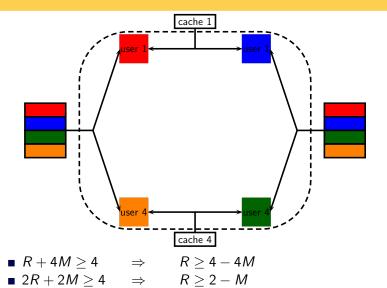


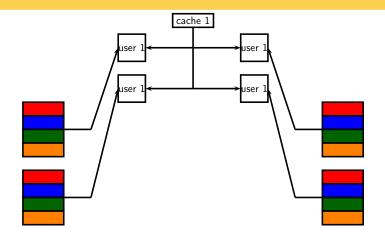




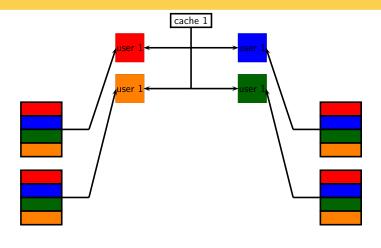
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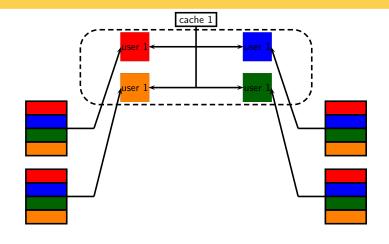




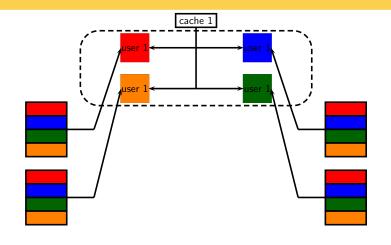
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■ This can be rewritten as

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For general N and K

$$R \ge \max_{s} \left(s - \frac{s}{\lfloor N/s \rfloor} M \right)$$

■ Comparing with achievable rate yields the theorem